Chapter 6 -- Resistive Stability

The NCSX stellarator will have a substantial amount, ≈ 100 - 200 kA, of plasma current driven inductively or by the bootstrap effect. The presence of plasma current provides a potential source of free energy which can then drive MHD instabilities such as tearing modes [6.1]. Calculations of the non-linear evolution of tearing modes, including neoclassical effects [6.2] (bootstrap current) in the full 3-D geometry of NCSX is beyond the capability of present MHD codes. In the tokamak community, qualitative modeling of tearing modes has been successfully done using a simple, quasi-cylindrical, low beta model as described below [6.3-6.8]. The validity of the application of this model to stellarators is supported by experiments on W7-A and W7-AS where reasonable agreement between experiment and modeling was found [6.9, 6.10]. The calculations presented below suggest that the start-up, equilibrium, and high beta phases of the baseline NCSX plasma should be stable to internally driven tearing modes. Neoclassical effects are predicted to further enhance this stability.

6.1 Present Understanding vs. Experiments

The Δ′ formalism used in the following analysis is derived in a zero beta, straight circular cylindrical geometry. Somewhat more refined formalisms (PEST III) allow for finite beta and shaping. However they are still constrained to axisymmetric equilibria and do not easily allow decoupling of ι (= 1/q) and J, nor do they calculate Δ′(w). Codes such as PIES or M3D can do a much more complete analysis, but are prohibitively expensive in terms of time to run. Tearing mode stability results found by application of the Δ′ formalism to shaped, finite beta and toroidally asymmetric plasmas must be viewed with some skepticism.

The Δ′ code used in the following calculations separates the ι(r) and J(r) profiles, necessary even in circular tokamaks such as TFTR, and particularly so in stellarators where a substantial fraction of the transform is not from the plasma current. The ι(r) and J(r) profiles are used in the standard differential equation governing the perturbed helical flux function [6.1]

$$[\partial^2/\partial r^2 + 1/r\ \partial/\partial r - m^2/r^2 - (\partial J_0/\partial r)/(\partial \psi_0/\partial r)]\ \psi_{m,n} = 0 \qquad (6.1.1)$$

where ψ_0 is defined from ι(r) by

$$\psi_0(r) = B_0/R_0 \int_0^r (\iota(r) - m/n)\ r\ dr\ . \qquad (6.1.2)$$

The J(r) includes the bootstrap, beam driven and inductively driven currents. Equation 6.1.1 has a pole at the mode rational surface where ι(r) = n/m. In the boundary layer region near this surface a full fourth order differential equation must be used; however it has been shown that the mode stability is determined by matching the external solution across the boundary layer using the "constant – ψ" approximation. The matching condition yields a discontinuity in the first derivative, which is quantified in Δ′. A positive value for Δ′ represents an unstable tearing mode, a negative eigenvalue stability.

This approach has been well studied and used extensively to analyze experimental tearing mode data in tokamak experiments. It provides both a basis for translating the measured external magnetic fluctuation levels into a measure of the island size as well as predictions of mode stability, growth rate and saturated island widths. In the circular cross-section TFTR tokamak this model found very good agreement between island widths predicted from edge magnetic fluctuation levels and island widths measured with the electron cyclotron emission temperature profile diagnostic [6.3].

Tearing modes have also been observed in stellarators such as the W7-AS and W7–A when net current is present. Simulations of the linear stability and non-linear evolution of the islands has been done, primarily with simple cylindrical Δ' models such as the one used here. In W7–A the analysis was also able to predict reasonably well the observed magnetic fluctuation level, *i.e.*, the saturated island width [6.9]. In the W7-AS experiment, these predictions were within an order of magnitude for the external magnetic fluctuation level and in reasonable agreement with the tomographically determined island size [6.10].

6.2 Δ' Analysis

Simulations of the start-up phase of the target NCSX plasma were done with the TRANSP code as described in Chapter 10. These simulations predict the evolution of the ohmic, beam driven and bootstrap current current profiles through the start-up phase to the target equilibrium. These time-dependent profiles have been analyzed for resistive stability to the (2/1), (5/3), (6/3), (7/3) and (7/4) tearing modes. The startup scenario has reversed shear (in the tokamak sense) and begins with $\iota(a) < 0.5$ [$q(a) > 2$]. As the plasma evolves the $\iota(a)$ drops until an $m = 2$, $n = 1$ rational surface enters the plasma from the edge at about 0.05 s.

The time dependent island width evolution is calculated by numerically integrating the generalized Rutherford equation [6.3, 6.5]

$$dw/dt = 1.22\ \eta / \mu\ [\ \Delta'(w) + \Delta_{nc}\ ]. \qquad (6.2.1)$$

where η is the resistivity and μ is the magnetic permeability. The $\Delta'(w)$ is calculated numerically using the constant-ψ approximation [6.11] and Δ_{nc} is evaluated by using parameters calculated by TRANSP in the equation

$$\Delta_{nc} = (16\ \pi / 5)\ k_1\ R_0\ J_{bs} / (s\ \iota\ B_0\ w\ ). \qquad (6.2.2)$$

Here J_{bs} is the local bootstrap current density, s is the local shear and w is the island width. The constant k_1 accounts for approximations made in deriving the effective perturbation in the bootstrap current due to the island. For the simulations shown here, the same $k_1 \approx 1$ was used as had been used to fit TFTR experimental data.

The results of such a calculation for the 2/1 mode are shown in Figure 6-1. In this simulation the neoclassical term was not included. The plasma is stable or marginally unstable throughout the start-up phase. The island width remains less than 1% of the plasma minor radius, which is inconsequential. Inclusion of the neoclassical term, which is stabilizing, reduces the island size even further, as discussed in Section 4.6.

The next lowest order modes are the 4/2, 5/3, 6/3, 7/3, and 7/4 modes. The 4/2, 6/3 and 7/3 modes were calculated to be robustly stable. The stability calculations for the 5/3 and 7/4 modes were problematic. For these modes, located near the plasma boundary, the relatively large local edge current density introduces strong curvature in the radial eigenmode structure. The appearance of the eigenfunction shapes suggests that this formalism is not applicable. The failure could either result from the mapping of non-axisymmetric, finite beta and shaped equilibria to a circular cross-section, quasi-cylindrical, zero beta model or might indicate that the plasma was nearing the ideal stability marginal point (known to result in similar problems even in the simpler tokamak axisymmetric geometry).

The TRANSP time-dependent simulation analyzed above was meant to reach the NCSX target "li383" equilibrium in steady state. There are some differences in the pressure and current profiles between the li383 equilibrium and the 38381w47 TRANSP run. A Δ' analysis was also done of the single time point li383 equilibrium. In this analysis the saturated island width reaches about 3.6 % of the minor radius without inclusion of neoclassical effects. With neoclassical effects the saturated island width is inconsequential ($\approx$ 0.2 %).

6.3 Neoclassical Tearing

The inclusion of neoclassical effects, i.e., the modeling of the effect of the island on the bootstrap current density and the concomitant effect of the perturbed bootstrap current on the island, has very successfully reproduced some of the observed characteristics of tearing modes in normal shear high beta, low collisionality plasmas. This extensive experimental database [*e.g.,* 6.3-6.8] gives some credence to the neoclassical tearing mode model. However, neoclassical theory (applied to tearing modes) in the context of reversed shear plasmas has not been extensively tested. The W7-AS experiments in which tearing modes were observed were reasonably well modeled without the inclusion of neoclassical effects. Whether the neoclassical terms would have qualitatively changed the results is not clear. The conclusion of the authors was that, "…, *so far no direct evidence of neoclassical effects on the stability has been found*." This statement could be interpreted as meaning there is no evidence either for or against the validity of the neoclassical theory of tearing modes. A study of double tearing modes in reversed shear plasmas in the TFTR tokamak also found no evidence for neoclassical modifications to the tearing mode stability in the negative shear regions [6.5]. However, in this case the analysis of double tearing modes was sufficiently unique that it is quite possible that the physics of the coupling in the double tearing modes was not adequately represented, leading to uncertainty in the conclusions. Further, single tearing modes were not observed in the reversed shear region of TFTR plasmas, consistent with the prediction of the neoclassical model that the bootstrap term is stabilizing in reversed shear.

6.4 Summary

The simulation of the NCSX start-up described in Chapter 10 has been analyzed for stability to tearing modes driven by ohmic, beam and bootstrap driven currents. The analysis has been done with a simple quasi-cylindrical Δ' code of the type used successfully in the analysis of tokamak plasmas. The plasmas are found to be stable to the low order tearing instabilities (4/2, 6/3, and 7/3 modes) and marginally stable to the 2/1 mode. The inclusion of neoclassical effects is generally believed to be stabilizing for plasmas with negative shear ($d\iota/dr > 0$, or $dq/dr < 0$), and the calculations suggest that the neoclassical terms result in a robustly stable 2/1 mode. The simple quasi-cylindrical stability calculations for the 5/3 and 7/4 modes located between $r/a \approx 0.85$ and the plasma edge did not give reasonable results, possibly indicating problems with the Δ' formulation or with the high local current density near the plasma edge.

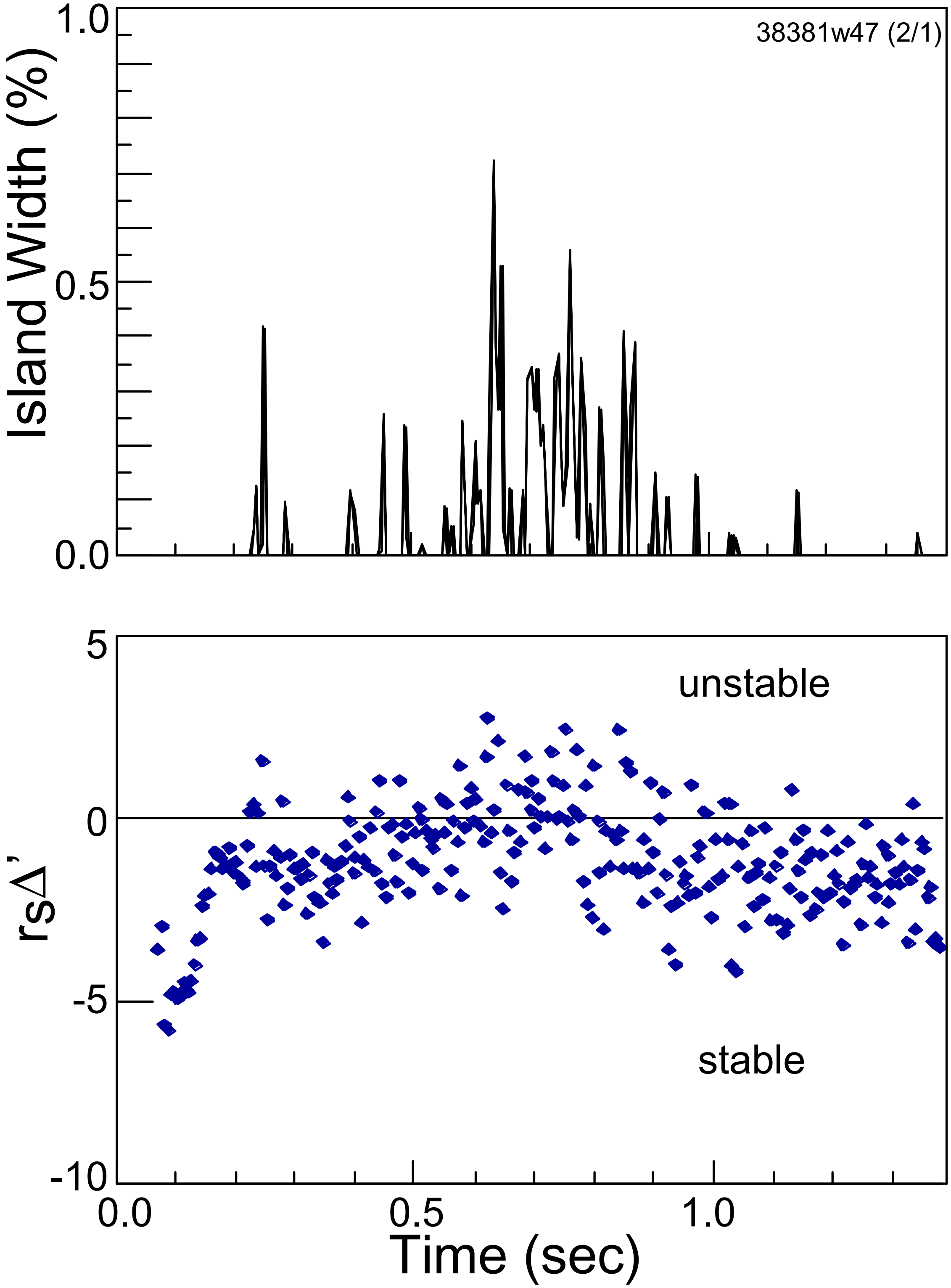
38381w47 (2/1)
Island Width (%)
1.0
0.5
0.0
5
0
-5
-10
rsΔ'
unstable
stable
0.0
0.5
1.0
Time (sec)

Figure 6-1

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