Reconstruction of Modular Coil Shape from Magnetic Field Measurement

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10/2004

Near Term Goals

- Demonstrate a method for accurately determining the modular coil winding center based on magnetic field measurement.
- Replace coil winding center representation with 3D coil model.
- Validate the procedure on the racetrack coil.

The shape of (as built) modular coils can be accurately determined from magnetic measurements

- We have developed both linear and nonlinear methods to reconstruct the shape of an NCSX modular coil from field measurements of either $|\mathbf{B}|$ or $[B_x, B_y, B_z]$.

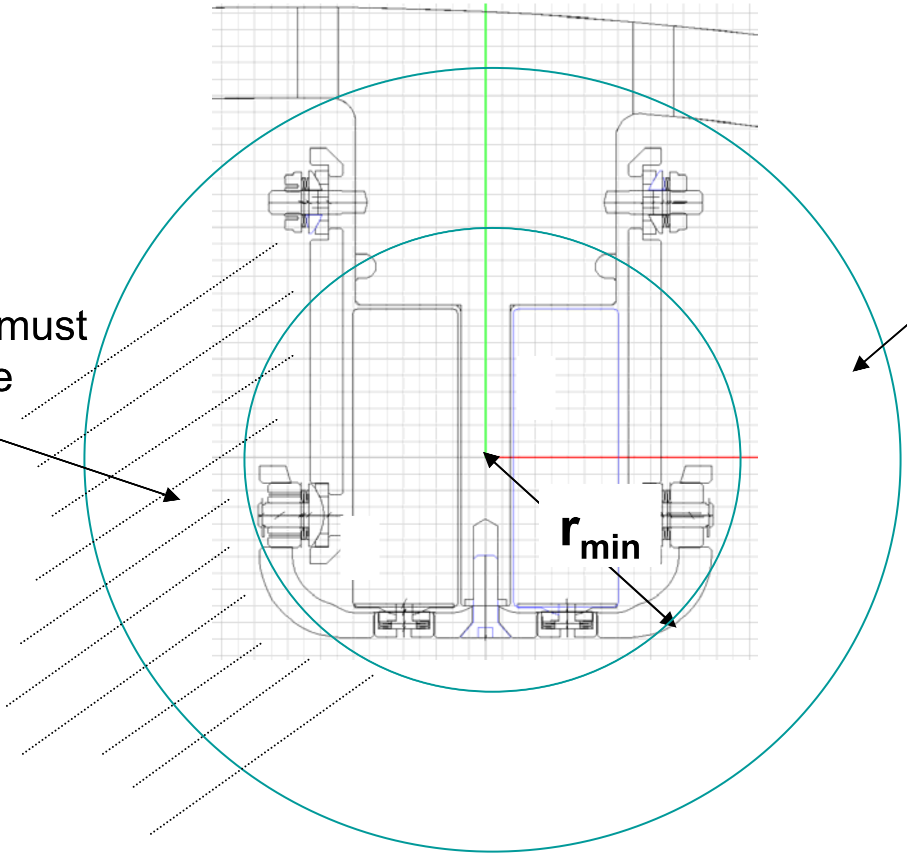
- It appears that a three axis hall probe, measuring components of the magnetic field in local coordinates, will work best for this application.

- 2000-3000 field measurements within 10-15 cm of the winding center are sufficient to reconstruct the coil centerline shape with average deviation of ≈ 0.05 mm (max. 0.25 mm) for sinusoidal distortions of magnitude 2 mm, and ≈ 0.1 mm (max. < 0.5 mm) for distortions of magnitude 5 mm, *assuming*
  - Field error in the magnetic measurements consistent with the hall probe technology,
  - Random error of magnitude ≤ 0.1 mm in the location of measurements.

Measurement locations are constrained by size and shape of the winding pack

Measurement region for data collection

Measurements must be taken outside coil case



3D annular region around design coil centerline used for testing reconstruction methods

Data will be collected by coupling a hall probe to a multi-axis measuring device

- Simultaneously measure magnetic field and location of measurement.
- NMR probes measure $|\mathbf{B}|$ very accurately, but have large sensing volume and require high field.
- Hall probe (www.gmw.com) measures B_x, B_y, B_z at lower field (10 – 2000 gauss) with sensing volume small relative to location error (±0.1 mm)

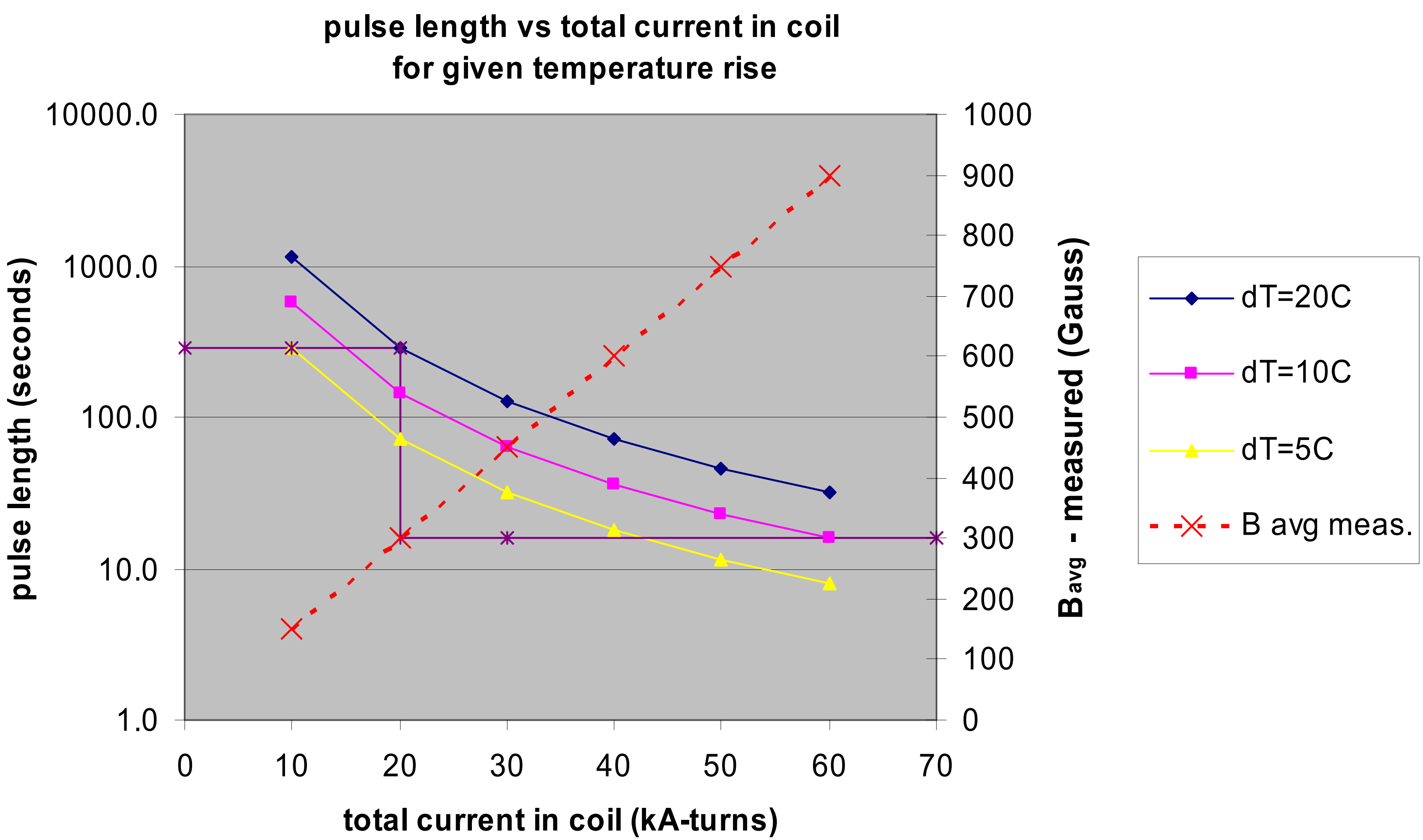
Accuracy of hall probe

- Components of field error:
  - offset error = ±1 gauss (can be subtracted from reading)
  - 0.1% of reading (= 0.001B)
  - noise component = ± 0.2 gauss

- Measurement location error:
  - position error of measuring device = ±0.1 mm
  - field sensitive volume = $[0.01\ \text{mm}]^3$

What are the limits on the coil for field measurement?

- Pulse length at ~ 300 gauss: 300 sec for 20 C temperature rise or 70 sec for 5 C rise



Numerical methods are compared for accuracy in shape reconstruction

- Linear approximation appears to give best results
  - Solve for corrections to design coil coordinates
  - Use singular-value decomposition

- Also considered nonlinear approximation methods
  - Parameterization of coil centerline
  - Fourier or cubic spline representation
  - Use Levenberg-Marquardt to find coefficients

For perturbations of prescribed phase and magnitude about the design coil shape, questions include:

- Number and location of measurement points?
- Is there sufficient accuracy in the field measurements and measurement location?

Procedure is tested on NCSX modular coil with known shape distortion

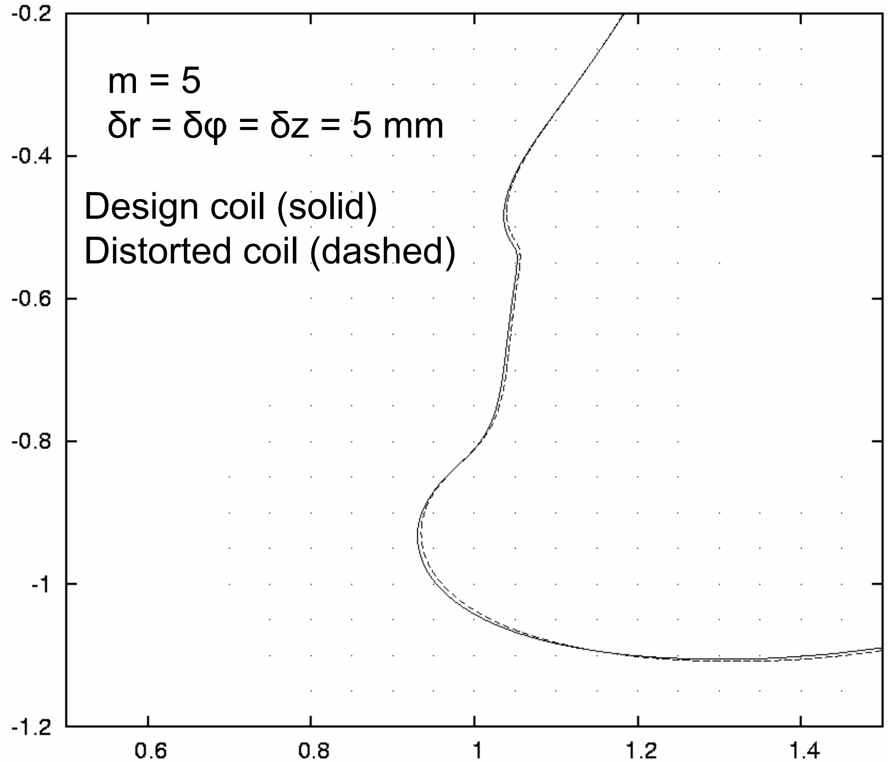
Sine, cosine coil distortions :

- $\Delta R = \delta r \sin(m\theta)$
- $\Delta\varphi = \delta\varphi \sin(m\theta)$
- $\Delta Z = \delta z \sin(m\theta)$

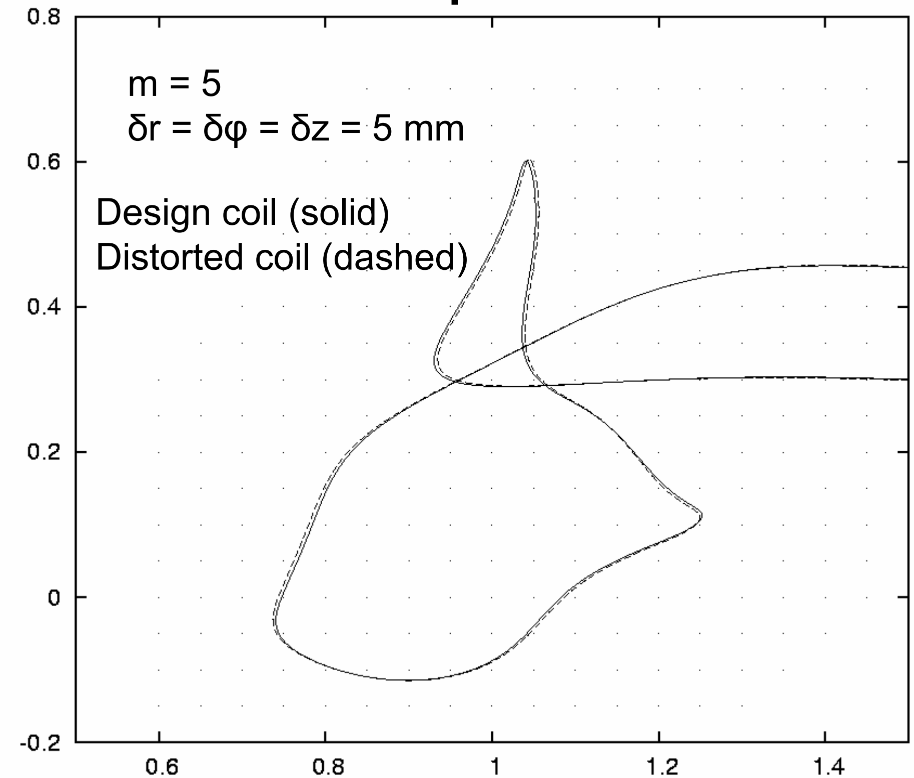
Random error included in:

- Magnetic field meaurement
- Location of measurements

Side view



Top view



Linear Method

Solve for coordinates of coil winding center

$\mathbf{x}_0 = [x_{01}, y_{01}, z_{01}, \ldots, x_{0n}, y_{0n}, z_{0n}]^T$	ideal (design) coil coordinates
$\mathbf{B}_0 = \mathbf{B}(\mathbf{x}_0)$	magnetic field of ideal coil over 3D grid
$\mathbf{x}_0 + \Delta\mathbf{x}$	coordinates of actual coil (manufactured)
$\mathbf{B}_1 = \mathbf{B}(\mathbf{x}_0 + \Delta\mathbf{x})$	magnetic field of actual coil (measured)
$\mathbf{B}_1 - \mathbf{B}_0 = \Delta\mathbf{b} \approx \mathbf{B}'\Delta\mathbf{x}$	$\mathbf{B}' = d\mathbf{B}/d\mathbf{x}$ is MxN Jacobian matrix
$\mathbf{B}' = \mathbf{U}\sum\mathbf{V}^T$	SVD
$\Delta\mathbf{x} = \sum_{\sigma n \neq 0} (\mathbf{u}_n^T \Delta\mathbf{b})\mathbf{v}_n / \sigma_n$	least-squares solution for $\Delta\mathbf{x}$

Drop ‘small’ singular values σ_n to stabilize solution, making it less sensitive to data vector $\Delta\mathbf{b}$

Distance between solution and distorted coil

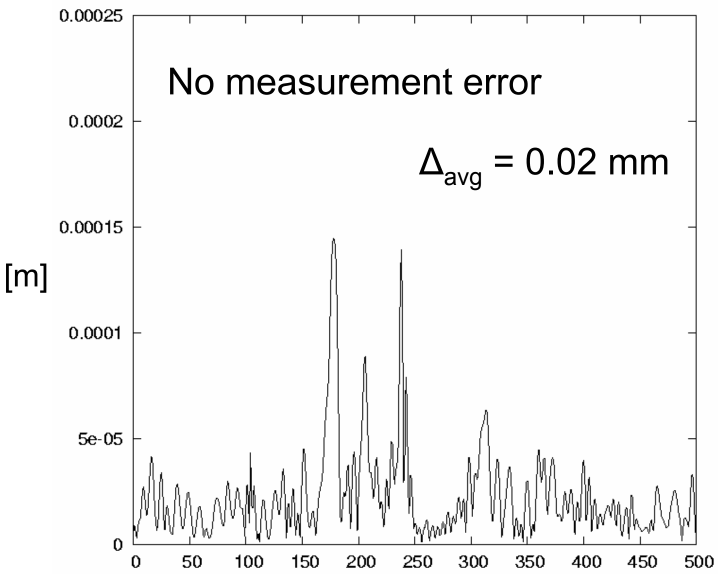
- Distorted coil represented by N=500 segments, unit tangent vectors $\mathbf{e}_\mathbf{i}^d$.
- Evaluate solution $\mathbf{x}$ (approximating field of distorted coil) at N points.
- For each point $\mathbf{x}_\mathbf{i}^d$ on distorted coil, find nearest solution point $\mathbf{x}_\mathbf{j}^*$, and perpendicular distance from solution point to distorted coil:

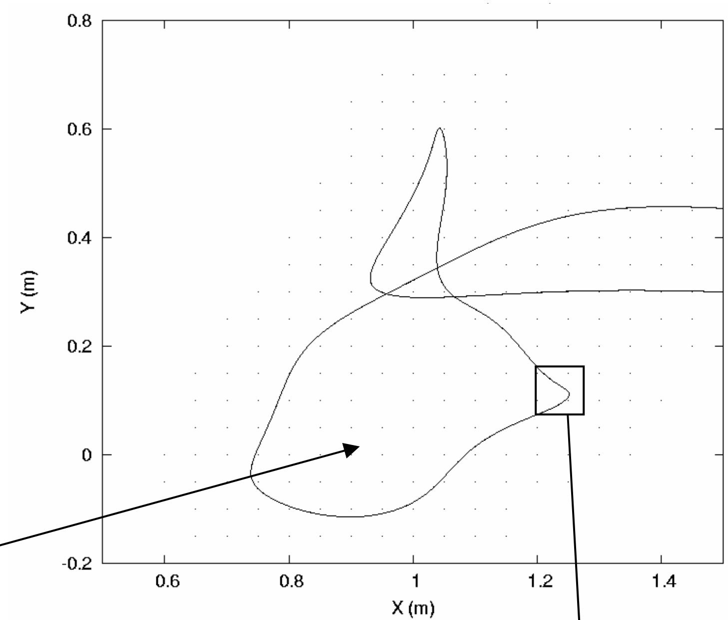
$$\Delta x_i = x_j^* - x_i^d, \qquad \Delta x_i^{\parallel} = [\Delta x_i \cdot e_i^d] e_i^d, \qquad \Delta x_i^{\perp} = \Delta x_i - \Delta x_i^{\parallel}$$

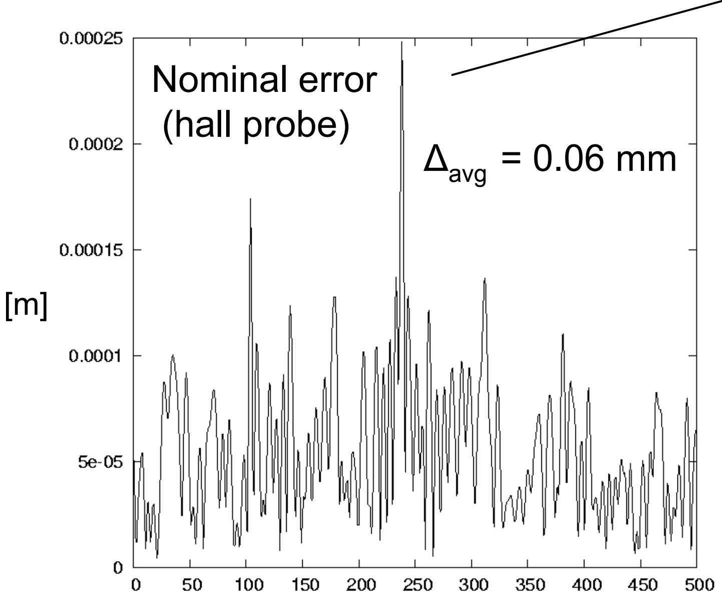
- Distance between solution and distorted coil:

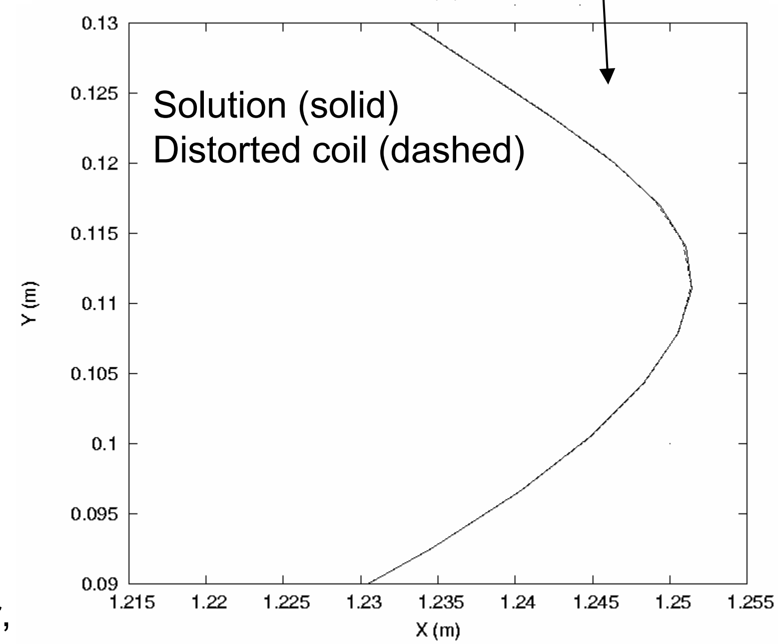
$$\Delta_{avg} = 1/N \sum_i |\Delta \mathbf{x}_i^{\perp}|, \qquad \Delta_{max} = \max \{ |\Delta \mathbf{x}_i^{\perp}| \}$$

Distribution of $\Delta x_i^{\perp}$ for winding center reconstruction

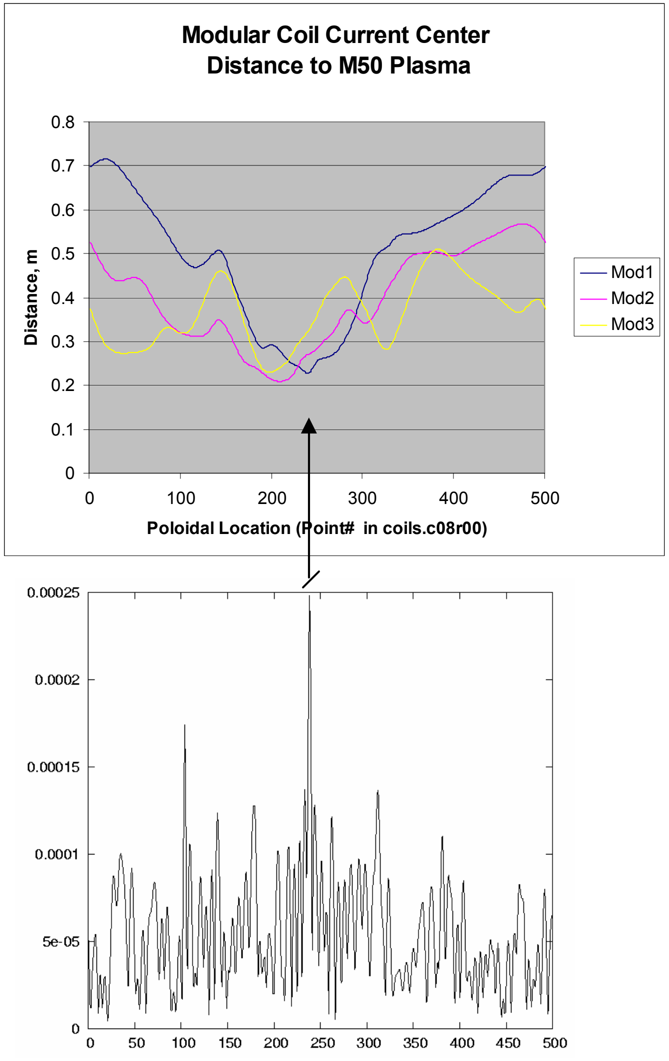








For linear method, Δ_{max} occures at points near plasma



Field measurement close to the coil improves accuracy in reconstruction of winding center shape

- sine coil distortion with m=6, $\delta r = \delta\varphi = \delta z = 2$ mm
- I = 20 kA; Field error: $|\delta B| \leq 0.2$ gauss + 0.1% of reading
- measurement location error ≤ 0.1 mm

Number of measurements	r_{min} [m]	Δ_{max} [mm]	Δ_{avg} [mm]
2250	0.10	0.240	0.051
2969	0.15	0.246	0.083
3495	0.20	0.827	0.121

For similar coil distortion and error in data, approximating magnetic field components gives better results than matching |B|

- Sinusoidal coil distortion, $\Delta R = \delta r \sin(m\theta)$, etc.., with m=6
- 2250 field measurements, $r \geq 10$ cm
- I = 20 kA; Field error: $|\delta B| \leq 0.2$ gauss + 0.1% of reading
- Error in measurement location ≤ 0.1 mm

Field match	δr, δφ, δz [mm]	Δ_{max} [mm]	Δ_{avg} [mm]
$[B_x, B_y, B_z]$*	2.0	0.145*	0.023*
|B|	2.0	0.404	0.133
$[B_x, B_y, B_z]$	2.0	**0.240**	**0.051**
|B|	5.0	0.913	0.191
$[B_x, B_y, B_z]$	5.0	**0.454**	**0.097**

* No field measurement or measurement location error

Avg. deviation between solution and distorted coil Δ_{avg} < 0.1 mm obtained for distortions up to 5 mm

Nonlinear Method

Solve for coefficients in parameterization of coil winding center

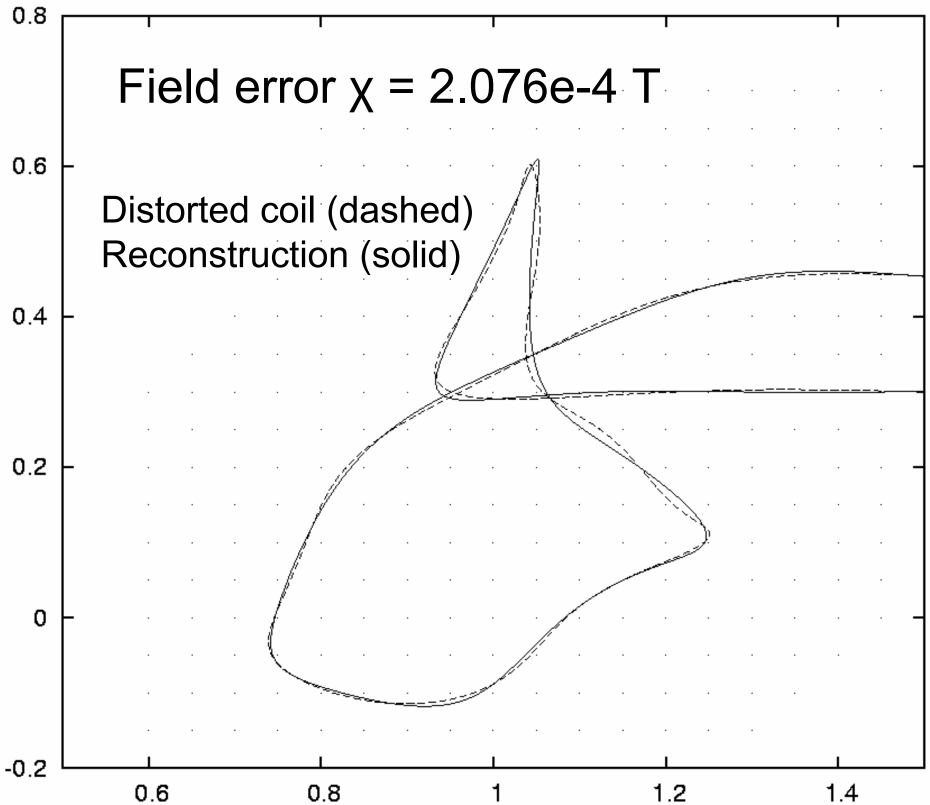
$\mathbf{x} = [x(t), y(t), z(t)]$	coordinates of coil centerline
$x(t) = a_{x0} + \sum a_{x,k}\cos(2\pi kt) + b_{x,k}\sin(2\pi kt)$, …	Fourier representation
$\mathbf{c} = [a_{x0}, a_{x1}, b_{x1}, \ldots, a_{zn}, b_{zn}]^T$	vector of coefficients
$\mathbf{B} = \mathbf{B}(\mathbf{c})$	magnetic field over 3D grid
Minimize $\chi^2 = \lVert \mathbf{B}(\mathbf{c}) - \mathbf{B}_1 \rVert^2$	$\mathbf{B}_1$ = field measurements on 3D grid

- Levenberg-Marquardt method to solve for coefficients
- Initial guess for c based on fit to design coil data
- Option in code for cubic spline representation: $x(t) = \sum c_{x,k} B_k(t)$, …

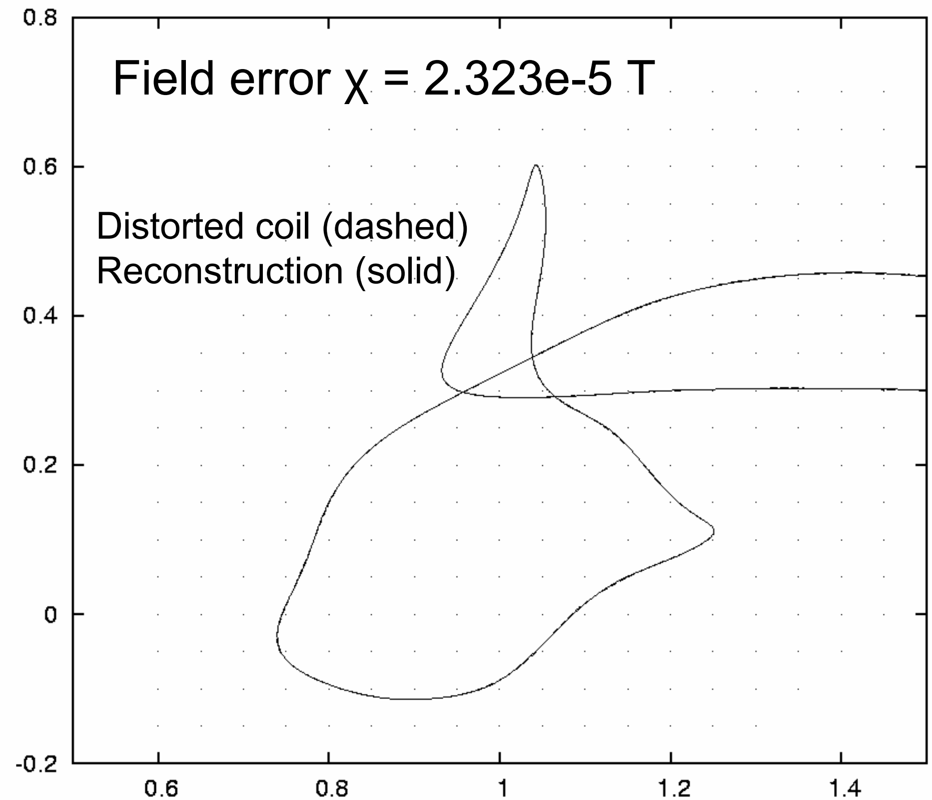
Nonlinear shape reconstruction of distorted modular coil requires > 150 of variable coefficients

• sin(5θ) distortion with δr = δφ = δz = 2mm

N_c = 81



N_c = 159



Differences between solution and distorted coil are slightly larger using the nonlinear method (compared to linear method) , for similar measurement error and distortion size

- sine coil distortion with m=6, $\delta r = \delta\varphi = \delta z = 2.0$ mm
- 2250 measurements, $10 \le r \le 15$ cm
- I = 20 kA, $|\delta B| \le 0.2$ gauss + 0.1% of reading
- Error in measurement location ≤ 0.1 mm

N_c	Δ_{max} [mm]	Δ_{avg} [mm]
159	1.015	0.451
315	0.309	0.113

Summary

- The “as built” NCSX modular coil winding center shape can be reconstructed with average error ≈ 0.05-0.1 mm for shape distortions of magnitude 2-5 mm and measurement error within limits of the hall probe technology.

- 2000 - 3000 field measurements within 10-15 cm of the (design coil) winding center are sufficient to reconstruct a coil centerline represented by ~500 segments.

To complete the task:

- Replace coil winding center representation with 3D (magfor) coil model.
- Transform field measurements in hall probe coordinates to coil coordinate system
- Test procedure on racetrack coil

Knowing the true shape of the coils, we can find the coil orientation and/or currents to optimize the vacuum field configuration

- Summary of recent STELLOPT modifications for vacuum field optimization
- Example

Module (VACOPT) added to stellarator optimization code (STELLOPT) to target resonances in vacuum magnetic field

- Minimize size of vacuum islands resulting from winding geometry errors by varying position of modular coils in array, or displacements due to magnetic loads / joule heating by varying coil currents.

- Additional STELLOPT variables include rigid-body rotations, shifts about coil centroid, and vacuum field coil currents.

- New targets include residues of prescribed resonances, bounds on variables, and constraints on position of the island O-points.

- Input list now contains poloidal mode numbers of targeted islands, initial values of vacuum field coil currents, shifts and rotations, and initial positions of control points for each modular coil.

- Output includes optimal coil currents and position of control points following optimal shifts / rotations of the modular coils.

Targeting Magnetic Islands in Vacuum Field Optimization

- Locate O-points of islands (order m fixed points of return map) $\mathbf{X}_i = (R_i, 0)$
- Following [1], compute residues of targeted islands

$$Res_i = [2 - \mathrm{trace}(T(\mathbf{X}_i))]/4$$

- Optimization

  Vary coil currents to minimize targeted residues:

$$\min.\ \chi^2 = \Sigma\, w_i Res_i^2,$$

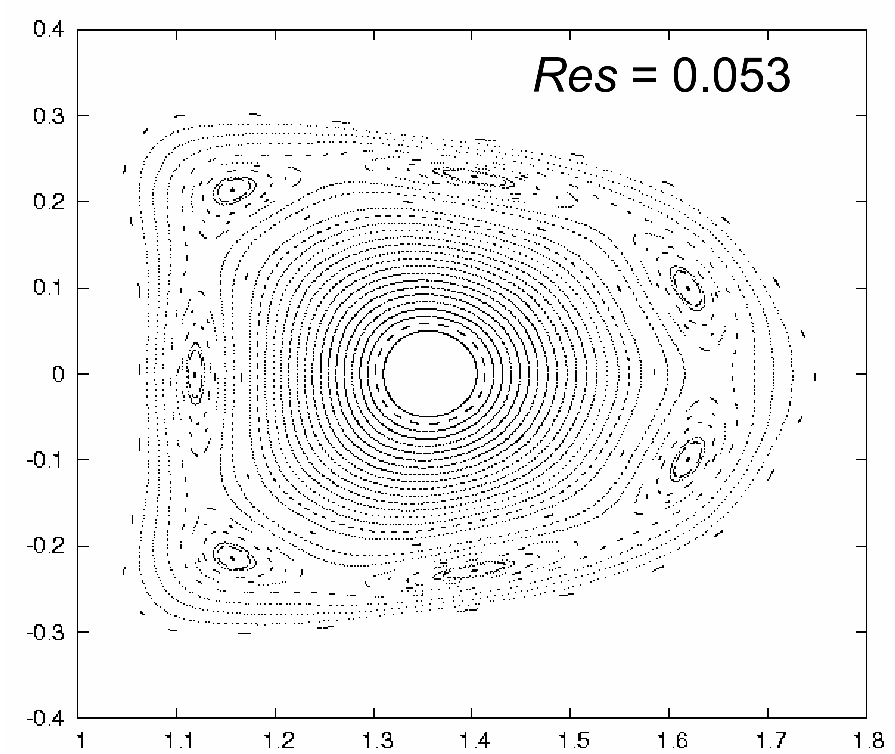
  Subject to possible constraints, e.g.:

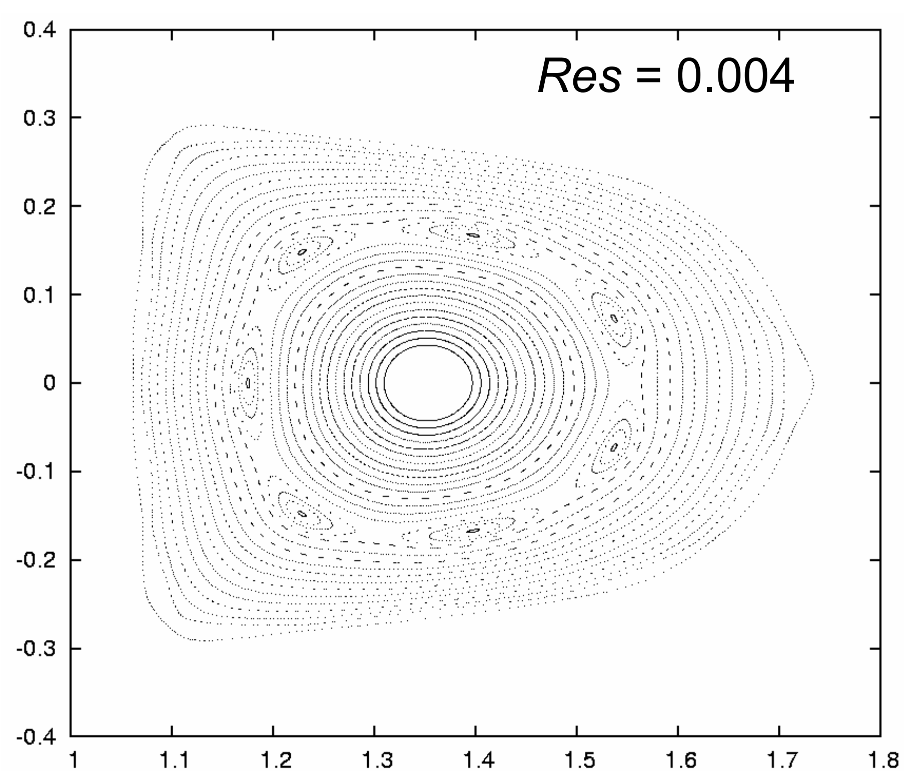
$$\Sigma\, I_{MOD} + I_{TF} = \text{constant}$$
$$R_{min} \leq R_i \leq R_{max}, \text{ (in prescribed toroidal plane } v = v_i)$$
Bounds on modular coil currents

[1] Cary and Hanson, Phys. Fluids 29 (8), 1986

Example: Vacuum field optimization by shifts and rotations about the centroids of modular coils

- Coil currents from 1.7T, high beta scenario at t=0.1s (vacuum)
- Minimize m=7 residue
- Constrain radius of O-point by R ≤ 1.18m (Z=0)





Example (cont'd): shifts and rotations about modular coil centroids for constrained minimization of the m=7 residue

	Δx (mm)	Δy (mm)	Δz (mm)	θ	φ	α *
M1	0.5	10.9	0.2	0.9866	-0.9471	0.0031
M2	3.0	-1.9	15.1	-0.0494	0.2878	-0.0058
M3	-0.7	-0.9	2.2	0.0669	0.0791	0.0056

* α measured with respect to unit rotation vector centered at coil centroid with sperical coordinate angles θ, φ.

Maximum changes in the control points are 1.17cm (M1), 1.58cm (M2), and 0.67cm (M3).

Summary

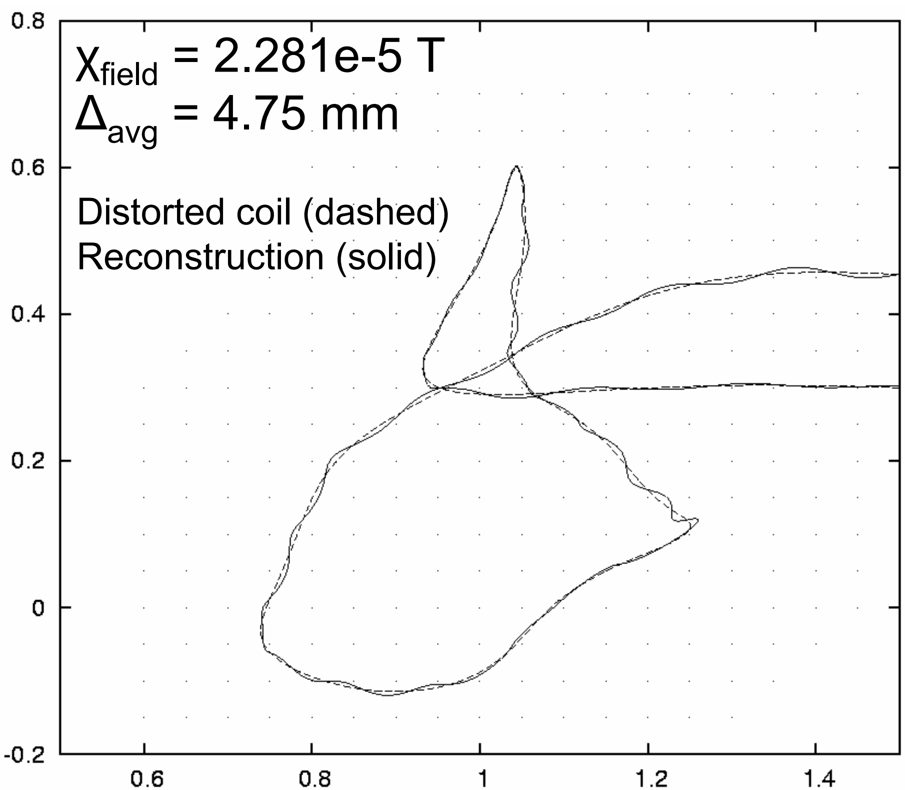
- Vacuum islands resulting from winding geometry errors may be minimized by varying position of modular coils in array
- Vacuum islands resulting from coil displacements due to magnetic loads / joule heating may be reduced in size by varying coil currents.
- Need to account for non-stellarator symmetric field errors

Additional Slides

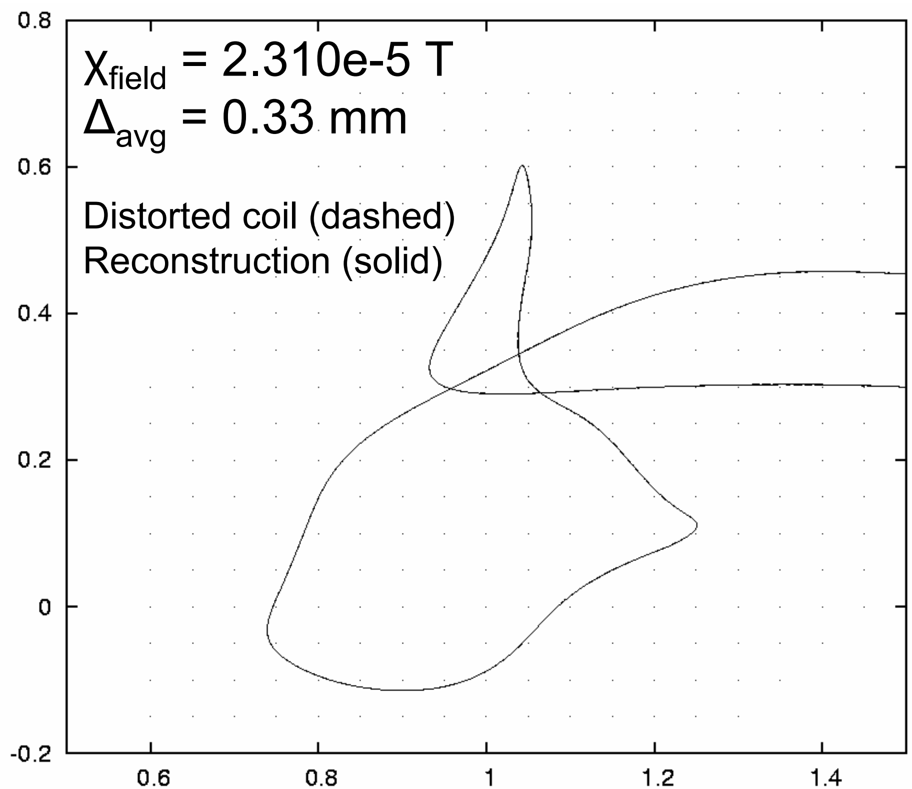
Quality of linear shape reconstruction depends on number of singular values retained (N_σ)

- Consider sin(5θ) distortion of modular coil M1 with δr = δφ = δz = 2 mm
- Random measurement error in field: $\delta B/B_{max} \le$ 1.e-5, location ≤ 0.1 mm

$\sigma_{min} / \sigma_{max}$ = 0.001, N_σ = 213



$\sigma_{min} / \sigma_{max}$ = 0.015, N_σ = 132



Linear coil shape reconstruction from magnetic field |B|

- sine coil distortion with m=6, $\delta r = \delta\varphi = \delta z = 0.002$ m
- 3495 measurements, $0.2 \text{ m} \le r \le 0.25$ m
- error in measurement location $\le$ 1.0e-4 m
- error in field measurement $\delta B/B_{max} \le$ 2.5e-4

$\sigma_{min} / \sigma_{max}$	N_σ	χ_{field} [T]	Δ_{avg} [m]
0.015	132	1.697e-5	4.268e-4
0.025	116	1.702e-5	2.464e-4
0.050	94	1.707e-5	1.503e-4
0.075	82	1.710e-5	1.494e-4
0.100	74	1.718e-5	**1.237e-4**
0.125	66	1.732e-5	1.527e-4

Field Line Equations and Tangent Map

•Magnetic field line is described by d**x**/ds = **B**(**x**)

•In cylindrical coordinates (and assuming $B_\varphi \neq 0$), these are reduced to two field line equations: $dR/d\varphi = RB_R/B_\varphi$, $dZ/d\varphi = RB_Z/B_\varphi$

•Integrating the field line equations, from a given starting point (e.g. in a symmetry plane), over a toroidal field period, produces a return map $\mathbf{X}' = M(\mathbf{X})$ ($\mathbf{X} = [R,Z]^t$)

•An order m fixed point of M is a periodic orbit: $\mathbf{X} = M^m(\mathbf{X})$

•The dynamics of orbits in the neighborhood of a fixed point are described by the tangent map: $\delta\mathbf{X}' = T(\delta\mathbf{X})$

(where $T_{11} = \partial M_R/\partial R$, $T_{12} = \partial M_R/\partial Z$, $T_{21} = \partial M_Z/\partial R$, $T_{22} = \partial M_Z/\partial Z$)