Pressure-Induced Breakup of Flux Surfaces in the W7AS Stellarator

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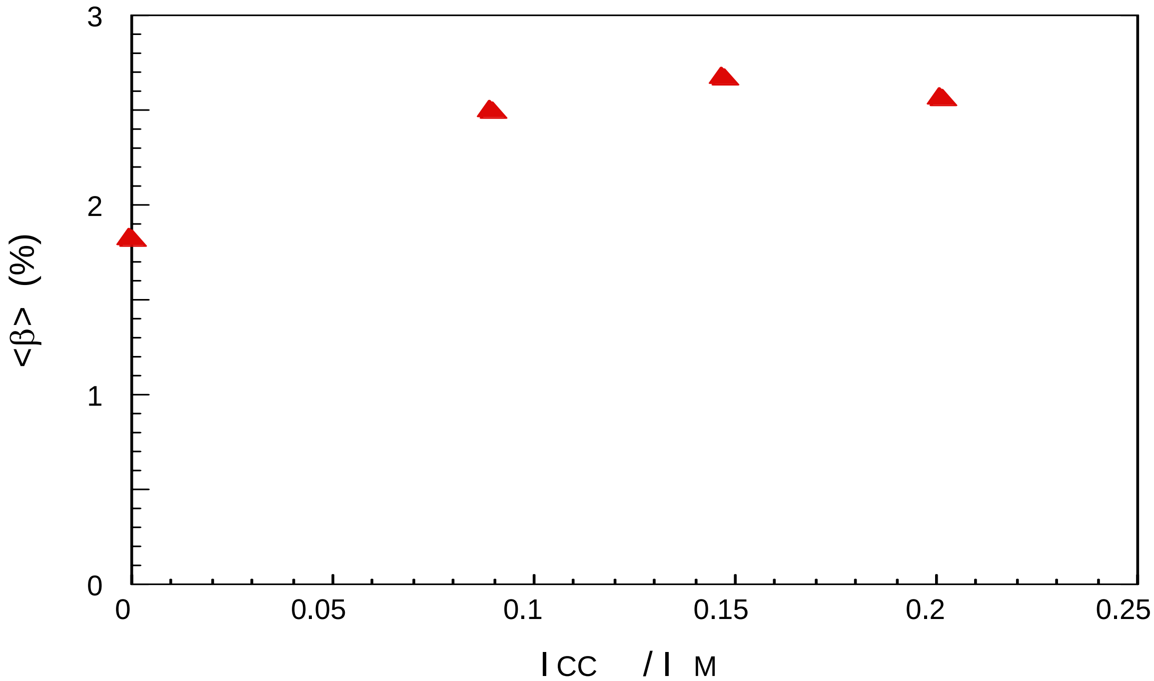
Introduction

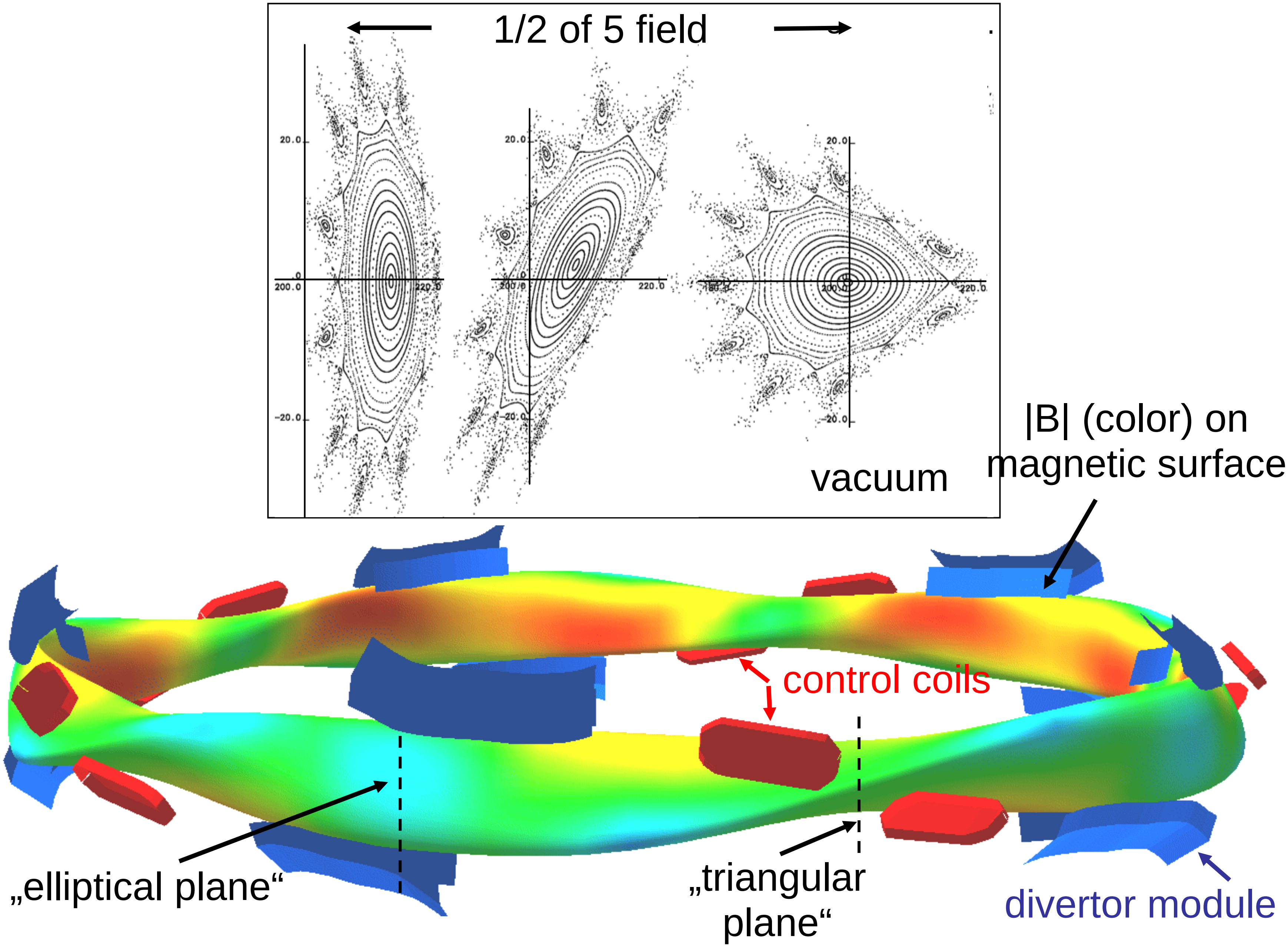
- An open question for many years: Are flux surfaces lost in high β stellarator equilibria? (L. Spitzer, Phys. Fluids **1**, 253 (1958).)
- Investigated in a number of theoretical papers:
    - Analytical calculations for simplified geometries.
    - Two equilibrium codes developed that can handle islands and stochastic regions: PIES (Princeton), HINT (NIFS).
- Only in recent years have stellarators reached sufficiently high β to study the issue experimentally. But no diagnostic tells us when surfaces broken.
- Compare code calculations with experimental observations for shots where observations suggest surfaces may be broken.
- This talk:
    - Calculation of equilibrium in stochastic regions;
    - PIES code calculations for two shots.

W7AS divertor control coil substantially affects achievable ⟨β⟩.



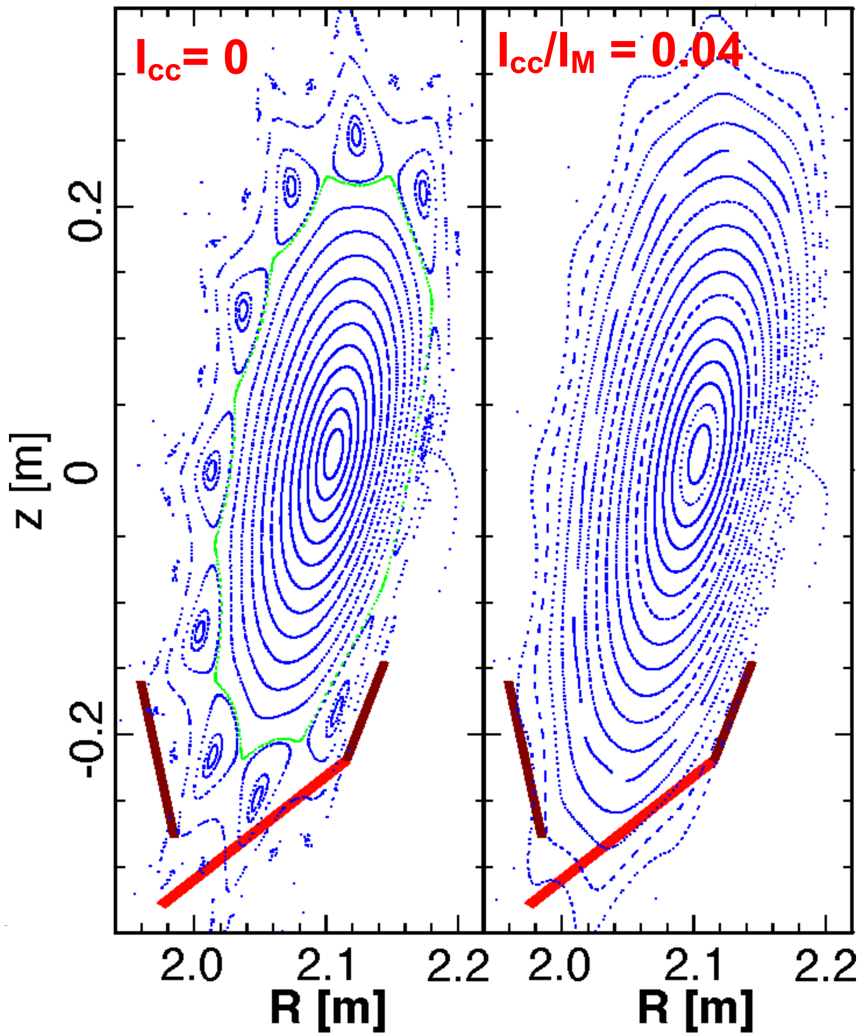
Variation of peak-⟨β⟩ versus the divertor control-coil current I_{CC} normalized by the modular coil current, for B=1.25 T, P_{NB} = 2.8 MW absorbed and ι_{vac} = 0.44.

Magnetic Configuration, In-Vessel Comp.



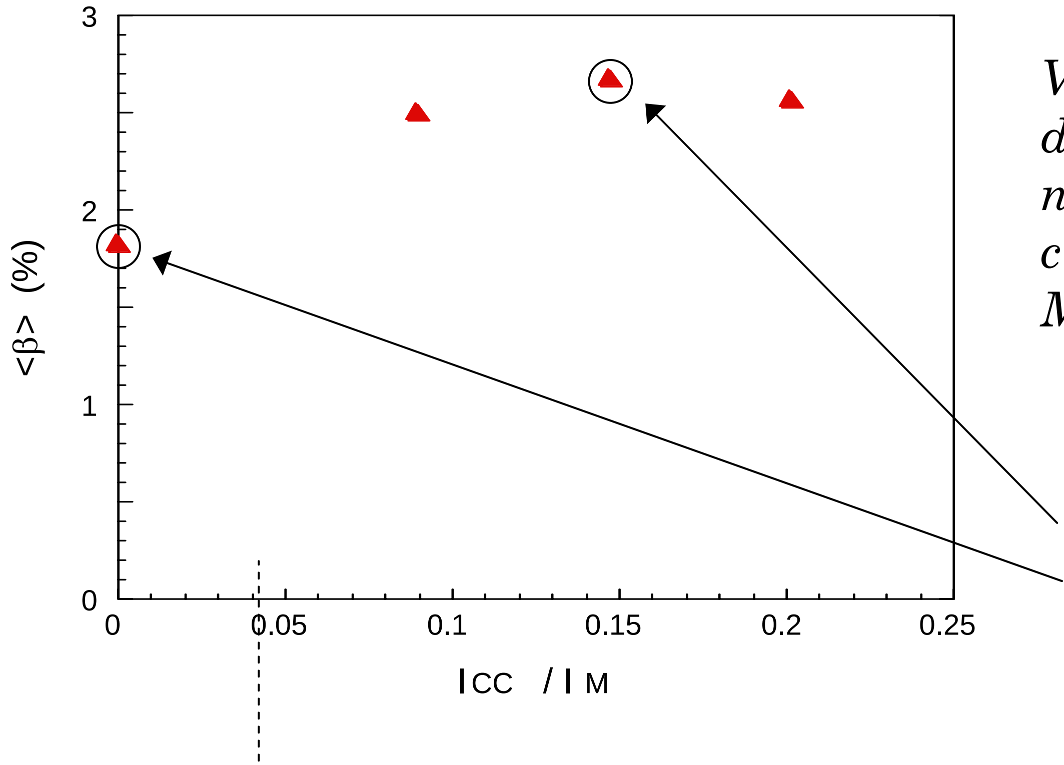
Divertor control coil on W7AS stellarator designed to control resonant magnetic field near plasma edge.

Calculated Vacuum Surfaces (Poincare Plots)



Coil calculated to have little effect on rotational transform ($\equiv \iota \equiv 1/q$), on neoclassical ripple transport, or on vacuum magnetic axis shift.

Optimal control coil current is about four times that producing good flux surfaces in vacuum field.

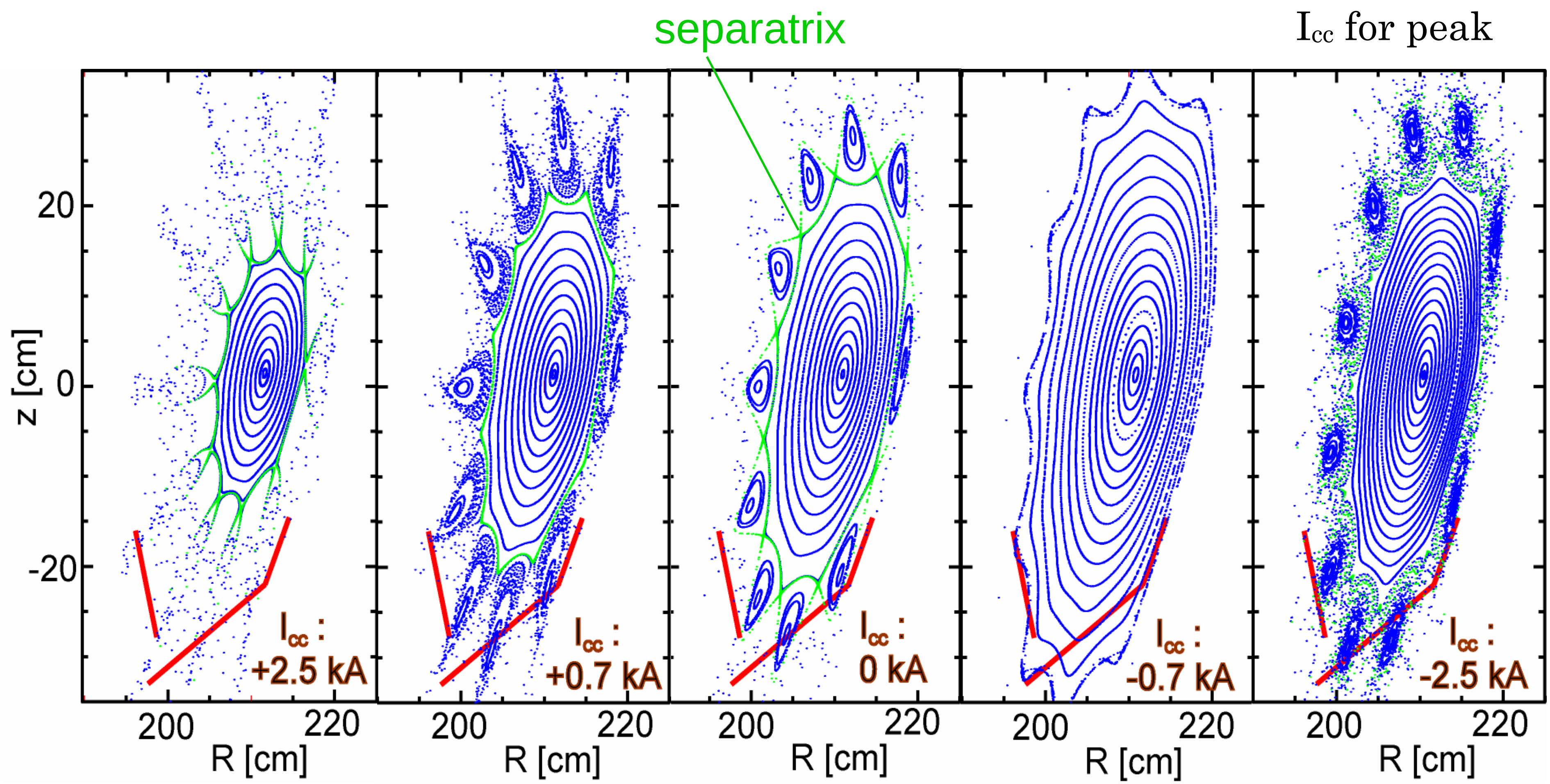


Variation of peak-⟨β⟩ versus the divertor control-coil current I_{CC} normalized by the modular coil current, for B=1.25 T, P_{NB} = 2.8 MW absorbed and ι_{vac}= 0.44.

PIES code run for these parameters.

Island width zero for vacuum field.

Effect of Control Coils
(Vacuum Configuration iota ≈ 1/2)



„Divertor-Configurations“

„High-β-Configurations“
(Divertor used as Limiters)

W7AS modeling uses experimentally determined pressure profiles (Zarnstorff)

- Optimizer used to construct VMEC equilibrium which provides best fit to experimental data.
- VMEC reconstructed equilibrium used as starting point for PIES calculation. Pressure profile preserved as code iterates.
- Initial calculations assumed zero net current in each flux surface. Recent work uses magnetic diagnostic info to reconstruct current profile.

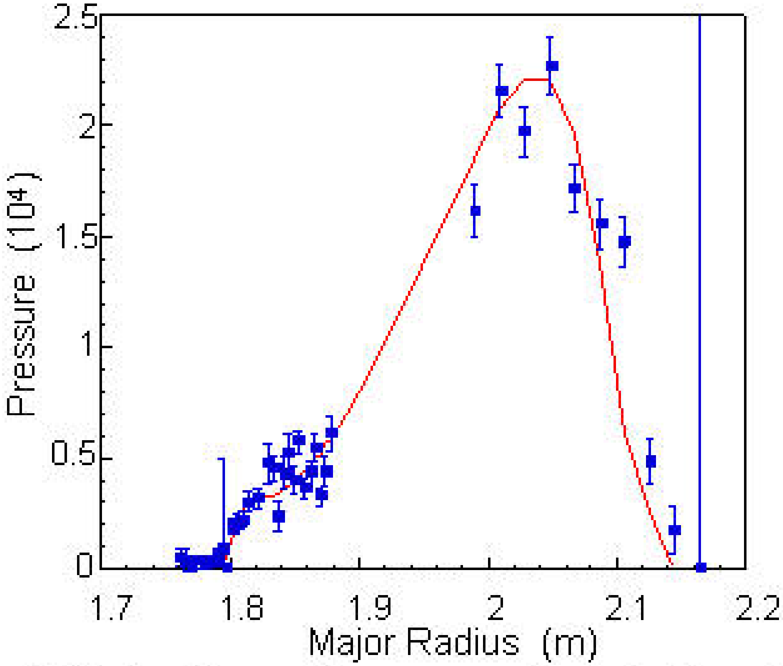
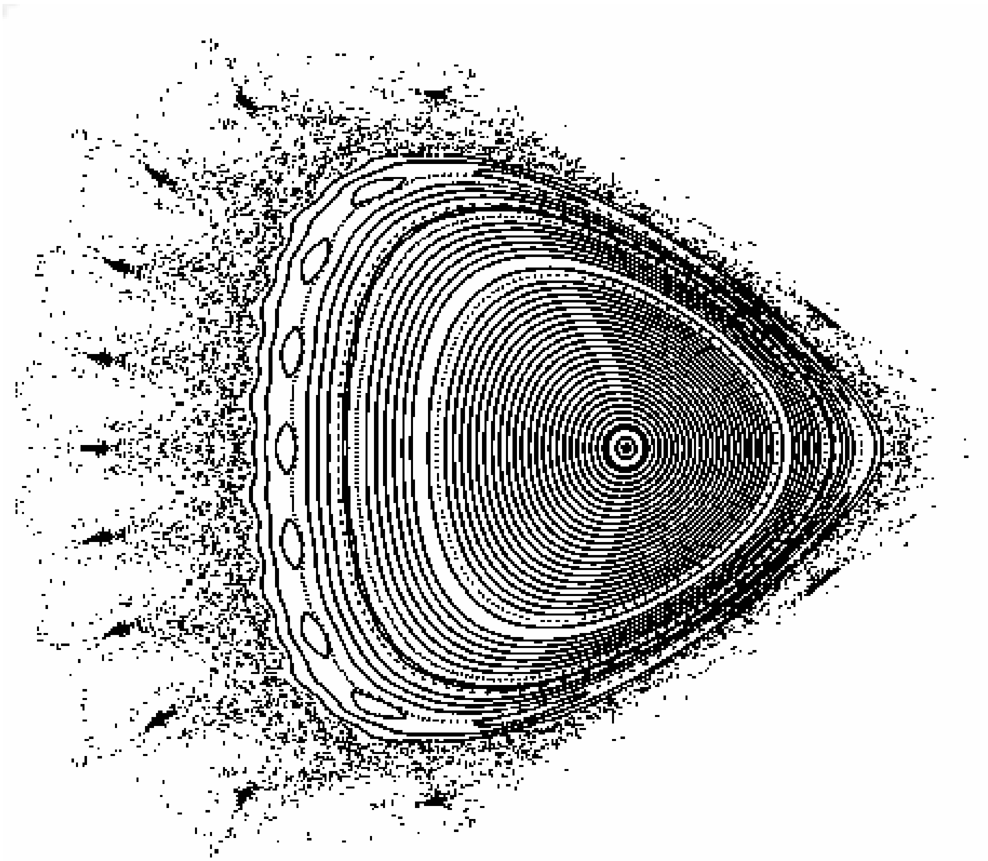


FIG. 4. Measured pressure profile (points) and fit profile from reconstruction (line) for plasma of Fig. 1. The data points are twice the measured electron pressure.

New version of PIES code keeps experimentally determined pressure profile fixed in stochastic region.



- PIES calculations find stochastic region at edge.
- Assume surfaces of constant pressure follow unperturbed flux surfaces, with field line stochasticity contributing to radial transport.
- Calculated width of stochastic region sensitive to pressure gradient in stochastic region.
- Need to solve equilibrium equations in stochastic region.

Equilibrium in Stochastic Regions

MHD equilibrium gives $\nabla p = 0$ in stochastic regions: $\mathbf{j}\times\mathbf{B} = \nabla p \Rightarrow \mathbf{B}\cdot\nabla p = 0$.

Consider tensor pressure, $\boldsymbol{P} = p\boldsymbol{1} + \boldsymbol{\pi}$, with $|\nabla\cdot\boldsymbol{\pi}| << \nabla p$.

Allow also weak flow, $|\rho\mathbf{v}\cdot\nabla\mathbf{v}| << |\nabla p|$.

$$\mathbf{j}\times\mathbf{B} - \rho\mathbf{v}\cdot\nabla\mathbf{v} - \nabla\cdot\pi = \nabla p$$

$$\mathbf{B}\cdot\nabla p = -\mathbf{B}\cdot(\rho\mathbf{v}\cdot\nabla\mathbf{v} + \nabla\cdot\pi) \quad \text{small}$$

Assume that flow along field lines adjusts to balance weak pressure gradient along slowly diffusing field lines.

Perpendicular force-balance gives closed, self-consistent set of equilibrium equations:

$$\mathbf{j}_\perp = \mathbf{B}\times\nabla p / B^2 + \mathbf{B}\times(\nabla\cdot\boldsymbol{\pi}+\rho\mathbf{v}\cdot\nabla\mathbf{v})/B^2 \approx \mathbf{B}\times\nabla p / B^2.$$

$$\nabla\cdot\mathbf{j} = 0 \Rightarrow \nabla\cdot(j_{||}\mathbf{B}/B) = -\nabla\cdot\mathbf{j}_\perp \Rightarrow \mathbf{B}\cdot\nabla(j_{||}/B) = -\nabla\cdot\mathbf{j}_\perp$$

$$\nabla\times\mathbf{B} = \mathbf{j}(\mathbf{B})$$

Have recast equilibrium equations in the form:

$$\mathbf{j}_\perp = \mathbf{B} \times \nabla p / B^2$$

$$\mathbf{B} \cdot \nabla (j_{||} / B) = -\nabla \cdot \mathbf{j}_\perp$$

$$\nabla \times \mathbf{B} = \mathbf{j}(\mathbf{B})$$

This is the form in which the PIES code solves 3D equilibrium equations.

Analogous to Grad-Shafranov Eq: $\Delta^* \psi = -R\mu_0 j_\phi = -\mu_0 R^2 dp / d\psi - FdF / d\psi$

- Can solve by Picard iteration: $\nabla \times \mathbf{B}^{(n+1)} = \mathbf{j}(\mathbf{B}^{(n)})$.
  More sophisticated scheme being installed: Jacobian-Free Newton-Krylov coupled to Levenberg-Marquardt for globalization.
- Given $\mathbf{j}(\mathbf{B}^{(n)})$, solve $\nabla \times \mathbf{B}^{(n+1)} = \mathbf{j}$ by matrix inversion.
  Finite differencing in radial direction, Fourier representation in θ and ϕ.
- 1D equation along field lines ("magnetic differential equation"):

$$\mathbf{B} \cdot \nabla (j_{||} / B) = -\nabla \cdot \mathbf{j}_\perp .$$

Need to solve along stochastic field lines.

Solution of Magnetic Differential Equation on Good Flux Surfaces

To solve $\mathbf{B}\cdot\nabla(j_{||}/B) = -\nabla\cdot\mathbf{j}_{\perp}$, transform to "magnetic coordinates". (Flux coordinates with straight field lines: $\mathbf{B}\cdot\nabla\psi = 0$; $\mathbf{B}\cdot\nabla\theta/\mathbf{B}\cdot\nabla\phi$ constant on flux surface = $\iota(\psi)$.)

Let $\mu \equiv j_{||}/B$.

$$\frac{\partial\mu}{\partial\phi} + \iota\frac{\partial\mu}{\partial\theta} = -\frac{\nabla\cdot\mathbf{j}_{\perp}}{B^{\phi}}, \quad B^{\phi} \equiv \mathbf{B}\cdot\nabla\phi.$$

Fourier decompose in θ and ϕ: $(nN - \iota m)\mu_{nm} = -(\frac{\nabla\cdot\mathbf{j}_{\perp}}{B^{\phi}})_{nm}$,

where n is mode no. per period, N no. of periods.

μ_{00} is constant of integration. Determined by profile of net current.

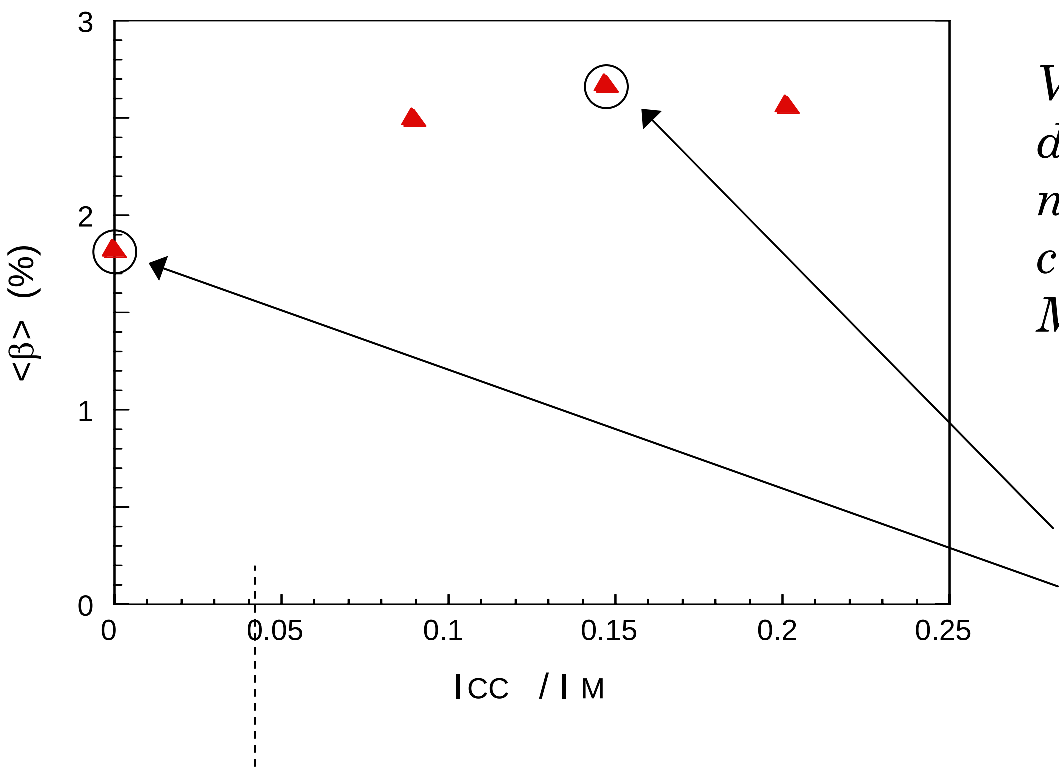
Mathematics of MDE in stochastic region is formally very similar to equations arising in turbulence resonance-broadening theory of Dupree and Weinstock.

- Perturbation solution of Vlasov equation has denominator with wave-particle resonance, $\omega - \mathbf{k}\cdot\mathbf{v}$.
- Incorporating effect of turbulent fluctuations in zeroth order gives integration along stochastically diffusing trajectory.
- Diffusion in velocity space moves particles with $\omega \approx \mathbf{k}\cdot\mathbf{v}$ in and out of resonance with wave, broadening resonances.

$$\frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}} \to \frac{\omega - \mathbf{k}\cdot\mathbf{v}}{(\omega - \mathbf{k}\cdot\mathbf{v})^2 + \Delta^2}$$

- Slow diffusion of trajectories has little effect on nonresonant Fourier components.
- Will return to MDE after examining structure of magnetic field in PIES stochastic regions.

We examine two W7AS shots which differ only in the magnitude of the current in the divertor control coil, but have very different values of experimentally attainable β (⟨β⟩ ≈ 2.7% vs. ⟨β⟩ ≈ 1.8%).



Variation of peak-⟨β⟩ versus the divertor control-coil current I_{CC} normalized by the modular coil current, for B=1.25 T, P_{NB} = 2.8 MW absorbed and ι_{vac}= 0.44.

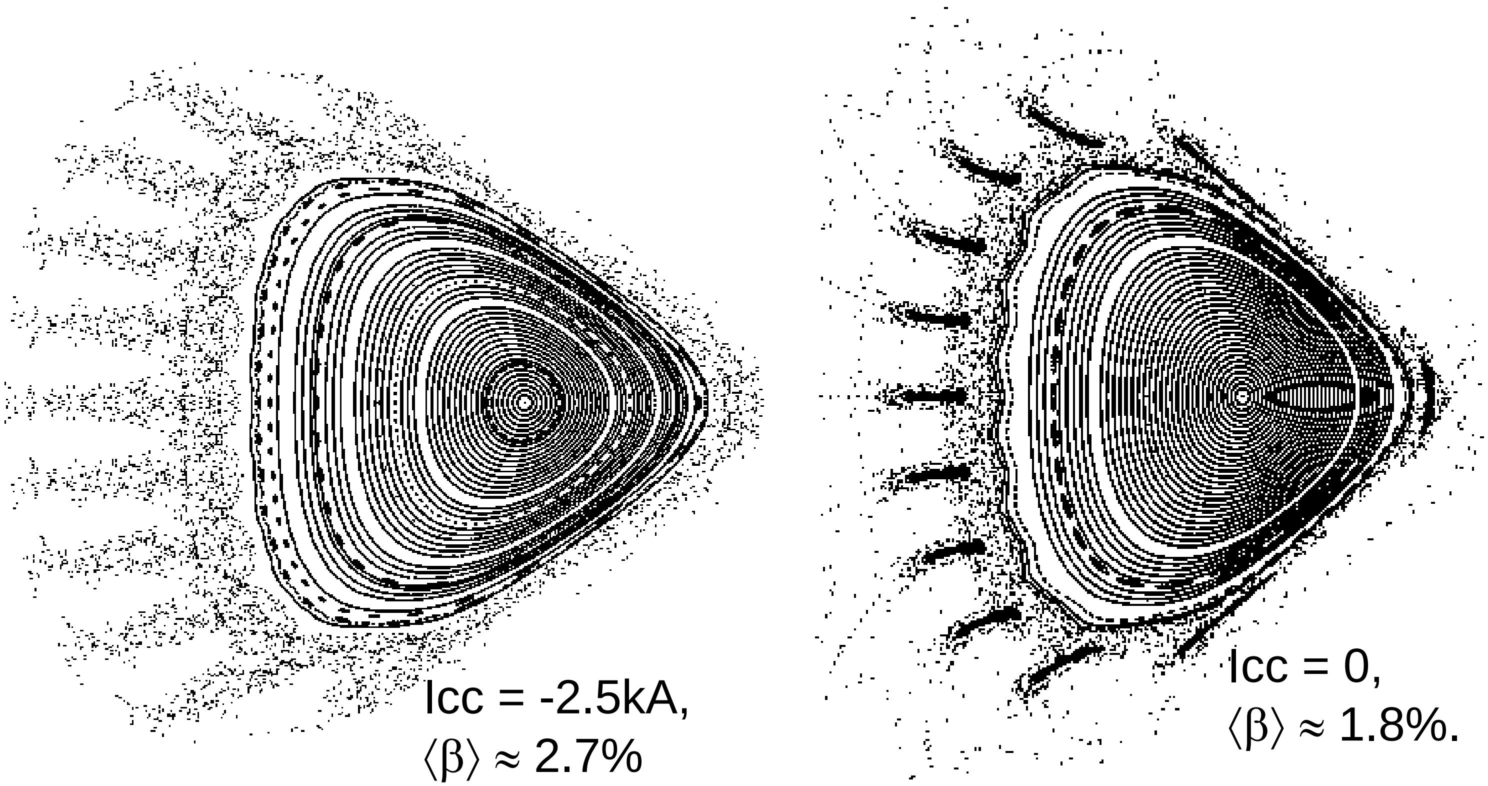
PIES code run for these parameters.

Island width zero for vacuum field.

Vacuum island width does not provide useful guide for choice of optimal I_{cc}.

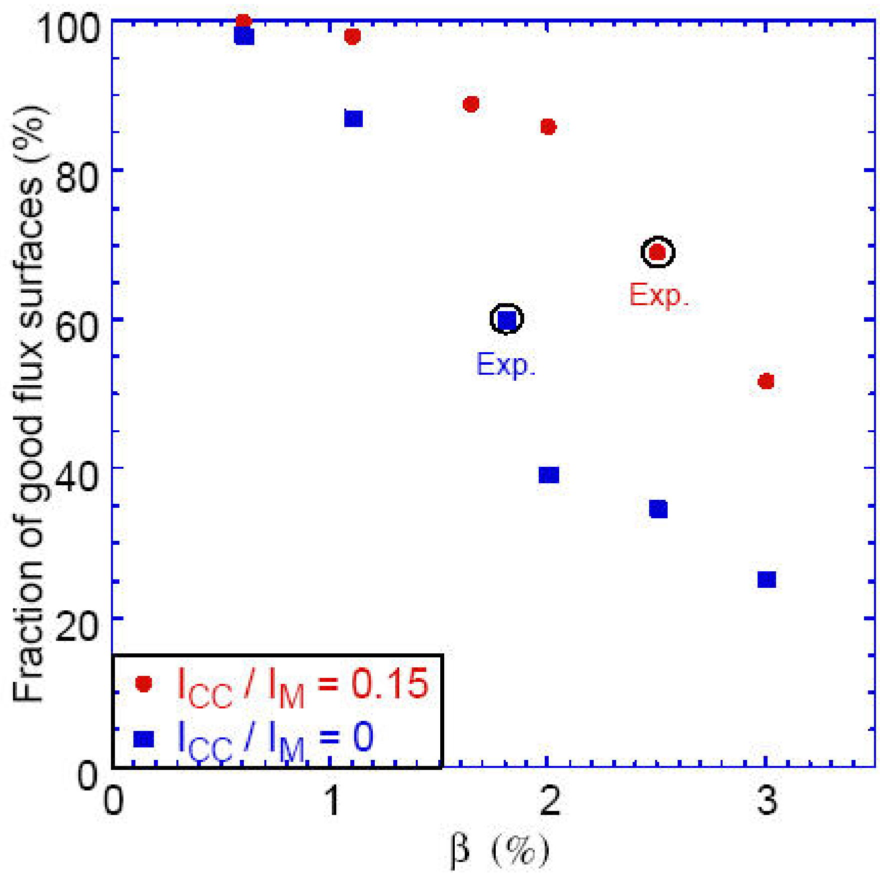
PIES equilibrum calculations find edge stochastic regions for both shots.

- Consistent with intuition that 3D surfaces broken at high ⟨β⟩ by compression and distortion due to Shafranov shift.



- Width of stochastic regions about the same, even though $I_{CC} = 0$ shot has substantially lower β.

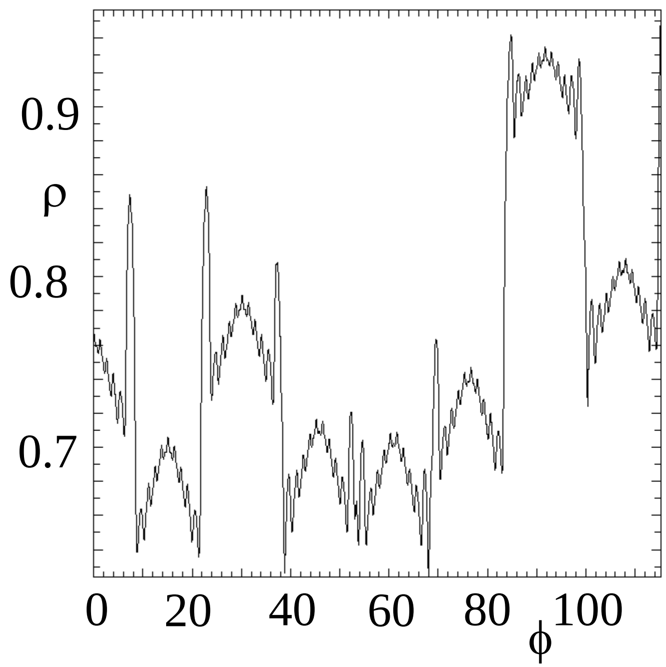
Calculations with zero net current found that width of stochastic region decreases with decreasing β.

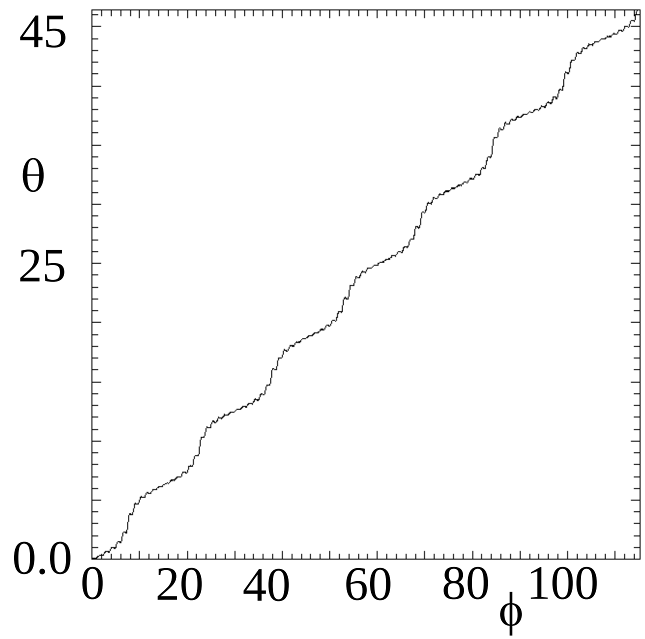


- Sequence of equilibria calculated for the two configurations with varying β and fixed pressure profile.
- Surfaces unbroken below β threshold.
- Above threshold β, equilibrium collapses if $\nabla p=0$ in stochastic region.

To study field lines in stochastic region, plot polar coordinates ρ and θ vs ϕ along field line. (Icc = -2.5kA, ⟨β⟩ ≈ 2.7% .)

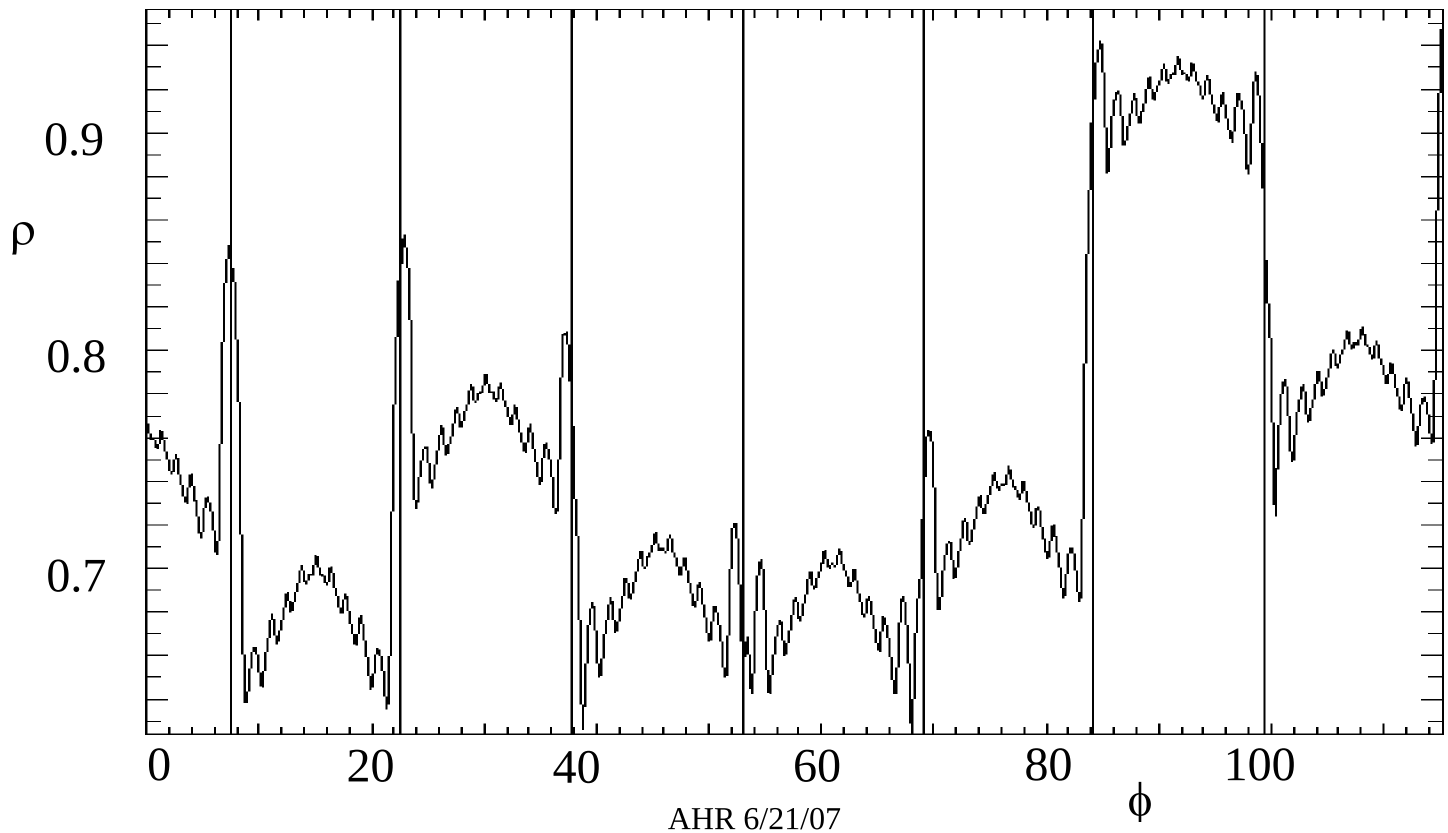
- ρ taken to be constant on VMEC flux surfaces, and to measure distance of flux surface from magnetic axis along outer midplane at ϕ=0. ρ ≡ 1 at edge. VMEC θ.
- ρ plot has localized oscillations punctuated by rapid, erratic radial excursions.



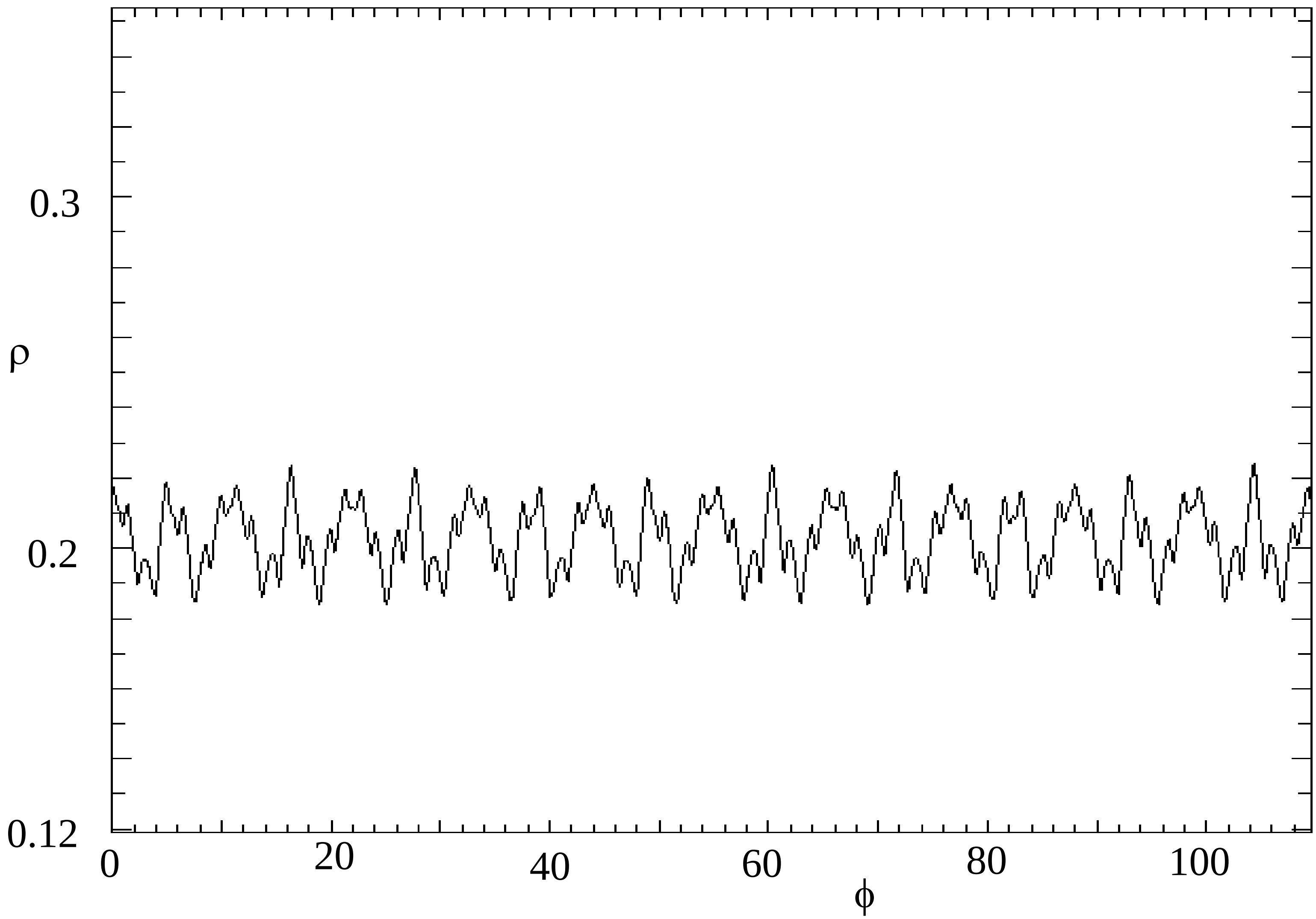


ρ plot with vertical lines at crossings of outer midplane shows that erratic radial excursions occur only near those crossings.

- Value of ρ between crossings returns to approximately its initial value.
- Field line behaves as if, in effect, flux surfaces in stochastic region punctured near outer midplane but remain intact elsewhere.



Plots of ρ on good flux surfaces do not have radial jumps.



Revisit Calculation of Pfirsch-Schlüter Currents in Stochastic Region

- $j_{||}$ determined by solution of $\mathbf{B}\cdot\nabla(j_{||}/B) = -\nabla\cdot\mathbf{j}_{\perp}$ along field lines.
- Field lines approximately follow unperturbed surfaces, except near outer midplane, where they execute erratic radial jumps.
- Can use magnetic coordinates, except near outer midplane:

$$\frac{\partial\mu}{\partial\phi} + \iota\frac{\partial\mu}{\partial\theta} = -\frac{\nabla\cdot\mathbf{j}_{\perp}}{B^{\phi}} = -\sum_{n,m}\left(\frac{\nabla\cdot\mathbf{j}_{\perp}}{B^{\phi}}\right)_{nm} e^{i(nN\phi - m\theta)}$$

where $\mu \equiv j_{||}/B$, $B^{\phi} \equiv \mathbf{B}\cdot\nabla\phi$.

- Take $\theta = 0$ on inner midplane. On a given field line, $\theta = \iota(\phi - \phi_0)$. Consider one Fourier mode at a time and superpose solutions.

$$\frac{d\mu_{nm}}{d\phi} = -\left(\frac{\nabla\cdot\mathbf{j}_{\perp}}{B^{\phi}}\right)_{nm} e^{i[(nN-\iota m)\phi + \iota m\phi_0]}.$$

Pfirsch-Schlüter Current in Stochastic Region (continued)

- For terms whose wavelength along field line is short compared to connection length, $nN - \iota m >> \iota$, get local solutions along field lines:

$$\mu_{nm} = -(\frac{\nabla \cdot \mathbf{j}_\perp}{B^\phi})_{nm} \frac{1}{i(nN - \iota m)} e^{i[(nN - \iota m)\phi + \iota m \phi_0]} + \mu_{nm0}(\phi_0),$$

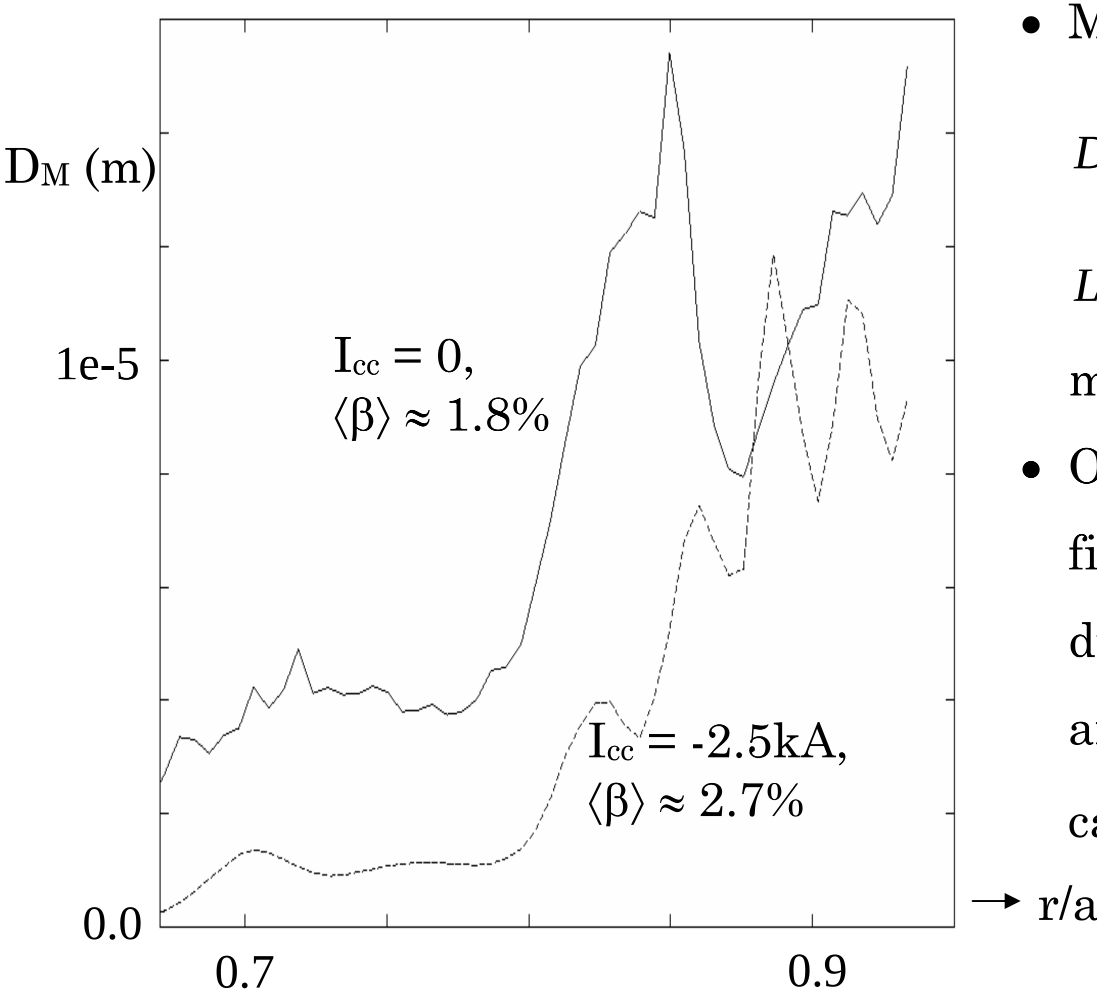
with corresponding solution on unperturbed surface,

$$\mu_{nm} = -(\frac{\nabla \cdot \mathbf{j}_\perp}{B^\phi})_{nm} \frac{1}{i(nN - \iota m)} e^{i(nN\phi - m\theta)}.$$

(Note that $\mu_{nm0}(\phi_0)$ does not contribute to this Fourier component on surface.)

- For $nN - \iota m << \iota$, wavelength along field line long compared to connection length, and phase mixing gives mode a small amplitude. Resonance broadening.

Calculated Magnetic Field Line Diffusion is Consistent with Differences in Achievable β



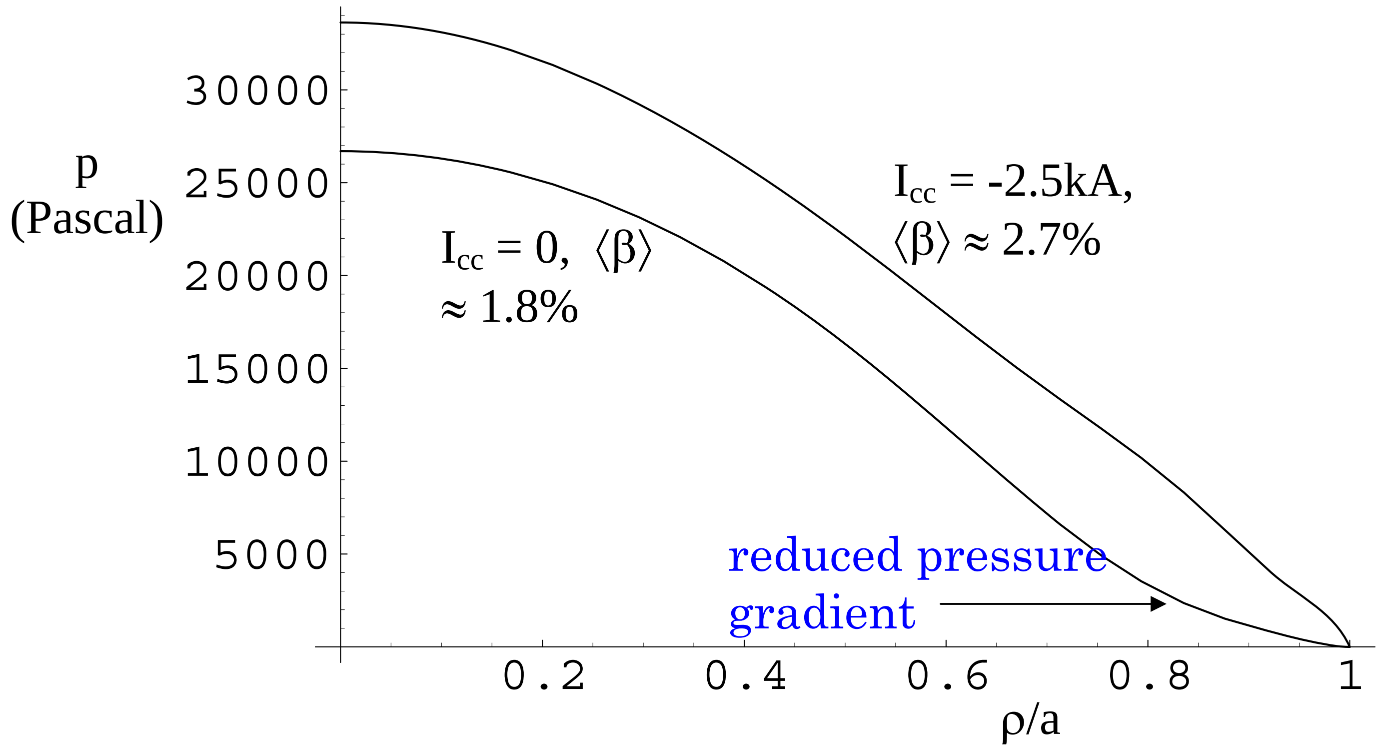
- Magnetic diffusion coefficient

$$D_M = \frac{\langle \Delta r^2 \rangle}{L_c} = \frac{a^2 \langle \Delta \rho^2 \rangle}{2\pi R / \iota},$$ where

  L_c is connection length, a is minor radius.

- On each surface, follow 100 field lines, launched at different values of ϕ, one time around poloidally, to calculate $\langle \Delta \rho^2 \rangle$.

Reconstructed pressure profiles consistent with picture that relatively large magnetic diffusion coefficient for I_{cc}=0 has deleterious effect on confinement.



Estimated contribution of stochastic field lines to energy transport consistent with expectation that effect substantial, but not sufficiently large to suppress pressure gradient.

- Energy carried along stochastic field lines by electrons.
  $\chi_{\text{stoch}} = a^2 \left\langle \Delta\rho^2 \right\rangle / \tau_c$, where a is minor radius, and τ_c is time for electrons to traverse a connection length.
- $\tau_c = L_c^2 / \chi_e$, where L_c is connection length and χ_e is diffusion coefficient for electrons along field lines.
- $\chi_e = (\text{v}_{\text{te}} / \nu_{ei})^2 \nu_{ei} = \text{v}_{\text{te}}^2 / \nu_{ei} \propto T_e^{5/2} / n_e$
- For I_{cc} = -2.5kA equilibrium at $\rho \approx .75$: $T_e \approx 200$ eV, $n_e \approx 2\times10^{20}$ m^{-3}, $\sqrt{\left\langle \Delta\rho^2 \right\rangle} \approx .04$. $\chi_{\text{stoch}} \approx .4$ m^2/sec. Sensitive to T_e.
- Planning to calculate Kolmogorov entropy and compare with Rechester and Rosenbluth (Phys. Rev. Lett. **40**, 38 (1978)).

Pressure-Induced Breakup of Flux Surfaces

- Intuition (See e.g. L. Spitzer, Phys. Fluids **1**, 253 (1958).): 3D flux surfaces broken by strong compression and distortion produced by Shafranov shift.

- Analytically tractable model (Reiman & Boozer, Phys. Fluids (1984).):
  - Large aspect ratio, nearly circular cross-section.
  - magnetic field = component with good surfaces + perturbation
  - Let (ψ,θ_m,ϕ_m) be magnetic coordinates for component with good surfaces. Breaking of flux surface determined by resonant component of $\mathbf{B}\cdot\nabla\psi/\mathbf{B}\cdot\nabla\phi_m$.
  - Express θ_m-θ, ϕ_m-ϕ as Fourier series in θ and ϕ. Strengthening of $\mathbf{B}\cdot\nabla\theta$ near outer midplane associated with Shafranov shift produces broadening of spectrum
  - Broadening of $\mathbf{B}\cdot\nabla\psi/\mathbf{B}\cdot\nabla\phi$ spectrum increases magnitude of resonant Fourier components.

Equilibrium Flux Surface Calculations for NCSX

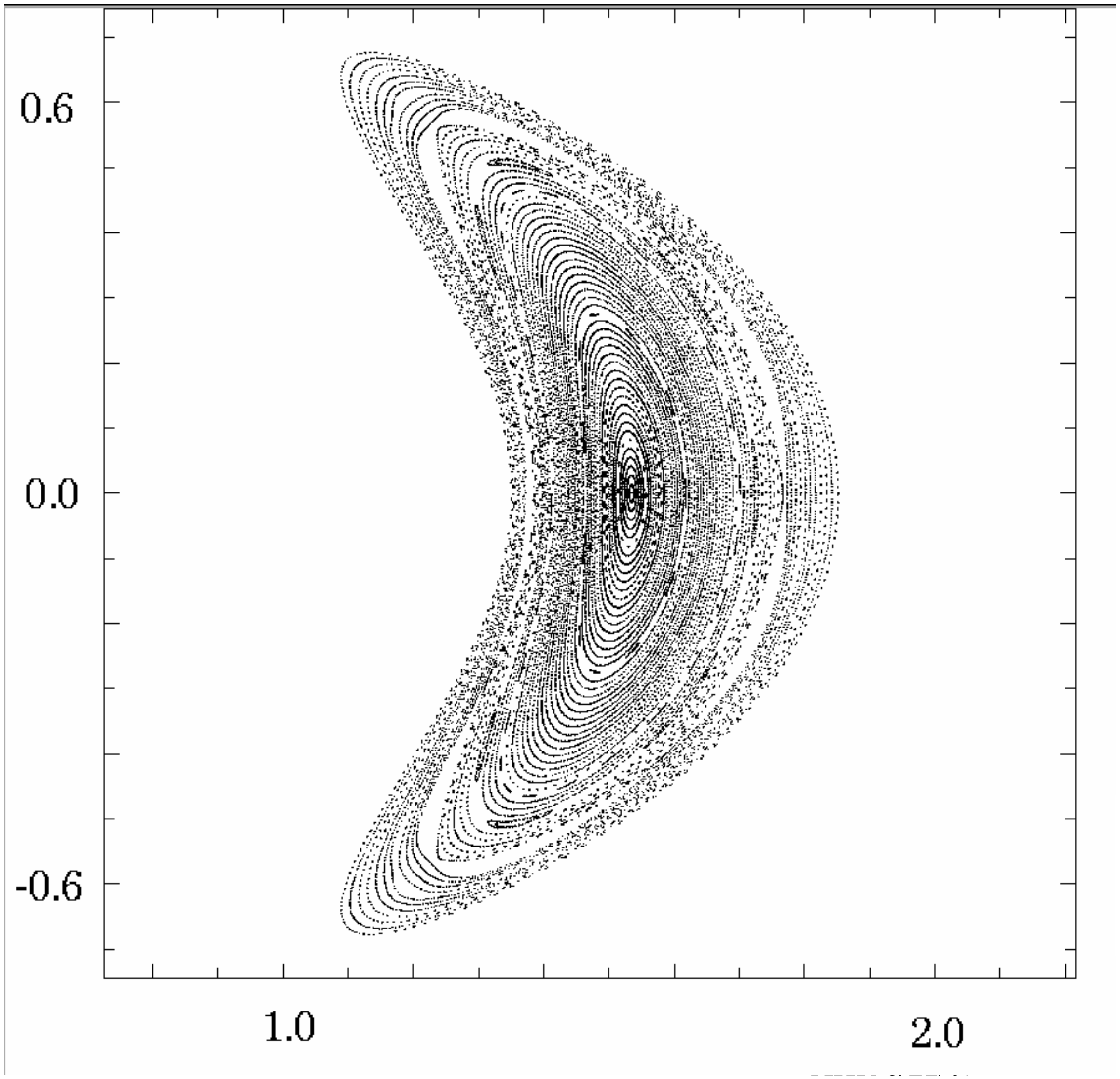
collaborators: S. Hudson, D. Monticello, L. Ku, M. Zarnstorff

NCSX: PIES code used as guide to design configuration with good flux surfaces.

1. PIES code used to identify fixed boundary configuration with intrinsically good flux surfaces.
2. Optimizer built around PIES code used to design coils which preserve flux surfaces, while also preserving desired physics properties of configuration and engineering properties of coils

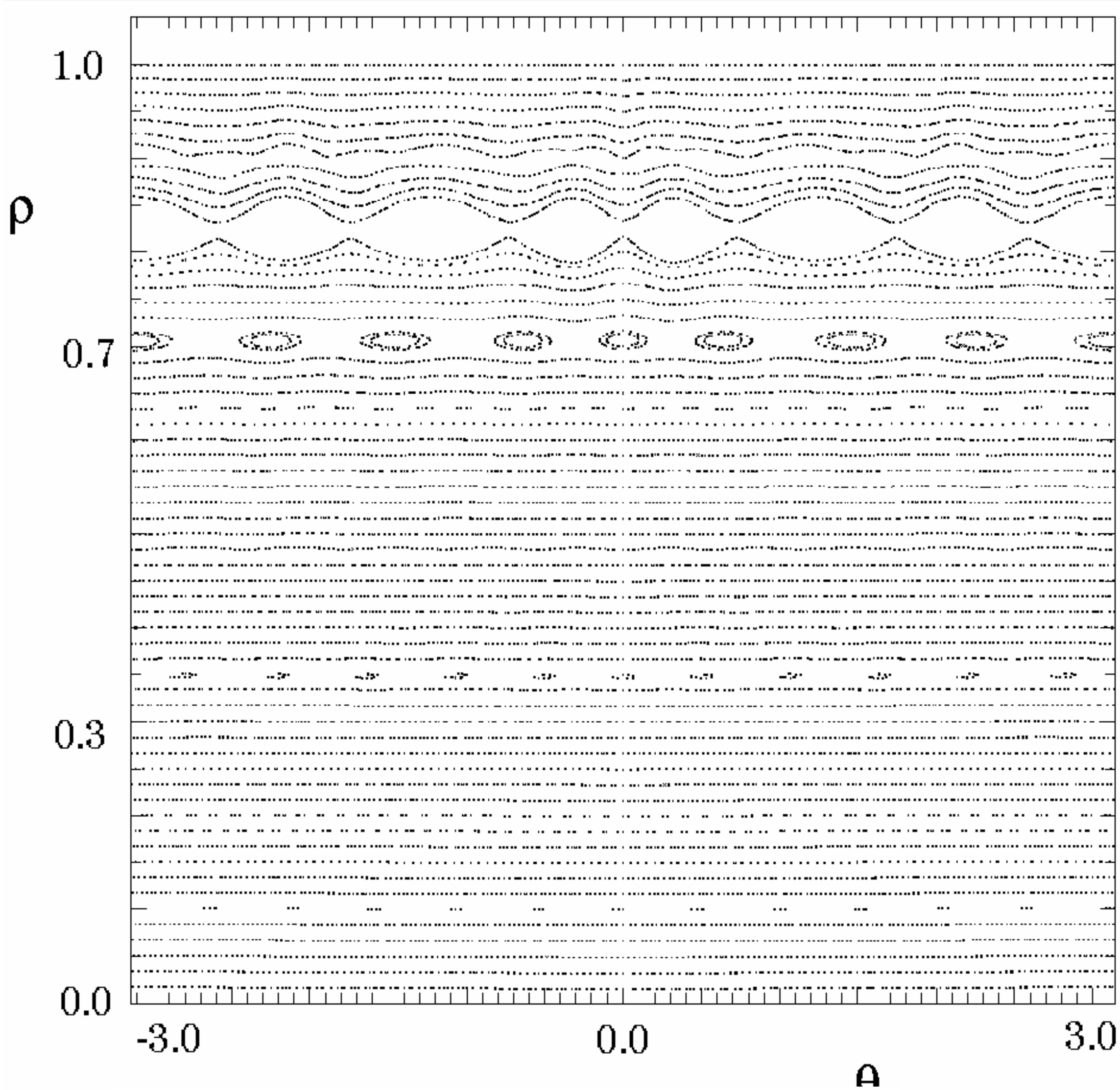
These calculations flatten the pressure gradient in the stochastic region.
Note: Stabilizing neoclassical effect not included in calculations.

PIES Calculations for NCSX: 1. Identification of Fixed Boundary Equilibria with Intrinsically Good Flux Surfaces



Poincare plot for an early candidate configuration at full current, $\beta = 0$. Shape of boundary specified ("fixed-boundary" equilibrium).

Early candidate configuration.



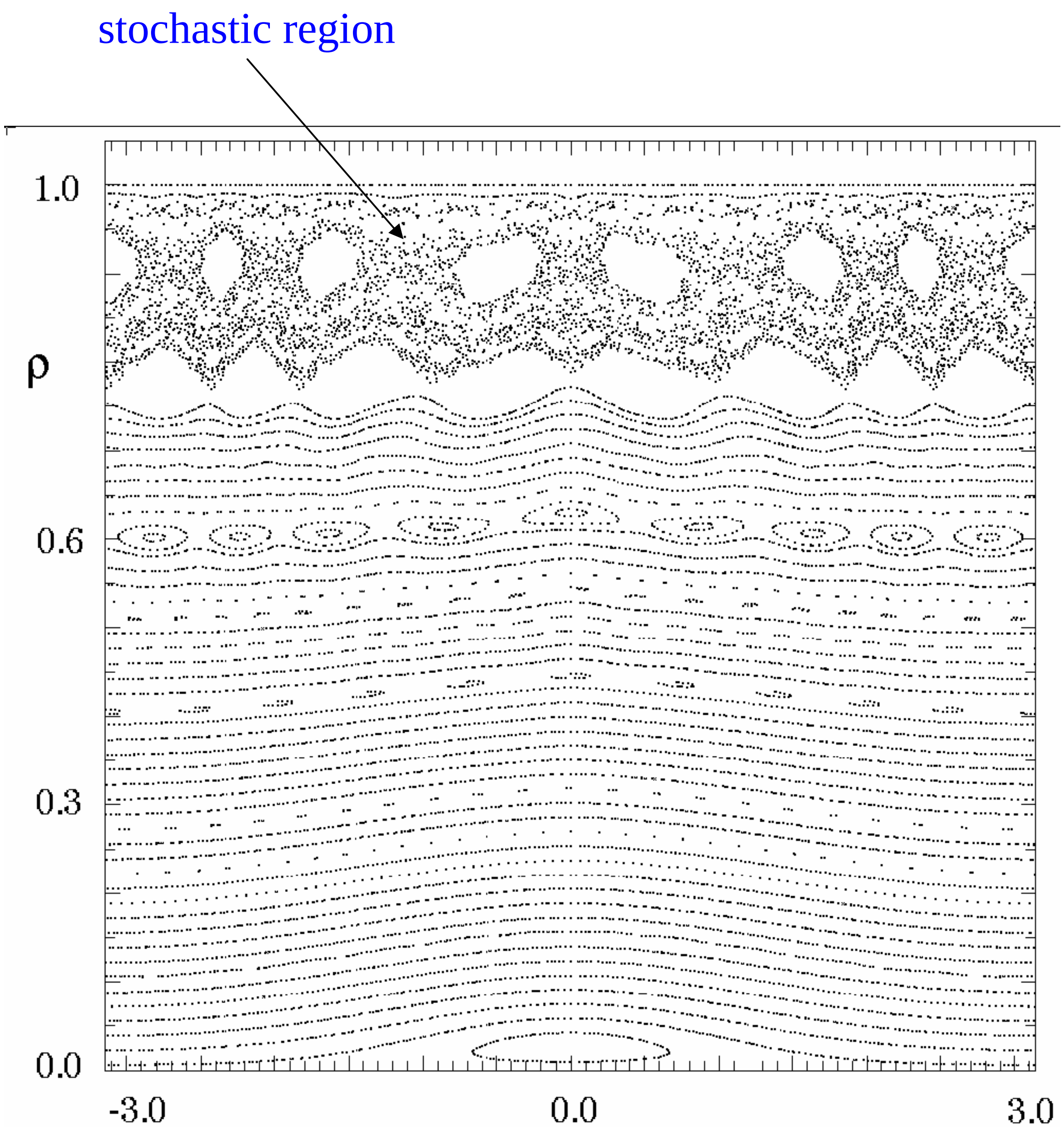
Early candidate configuration.

Plotting same data in polar (ρ, θ) coordinate system makes islands more readily visible.

Coordinate ρ taken to be constant on VMEC flux surfaces, and to measure distance of VMEC flux surface from magnetic axis along θ = 0, ϕ = 0 line.

Angular coordinate θ identical to VMEC angular coordinate.

Plotted in these coordinates, Poincare plot gives straight lines when VMEC and PIES solutions coincide.

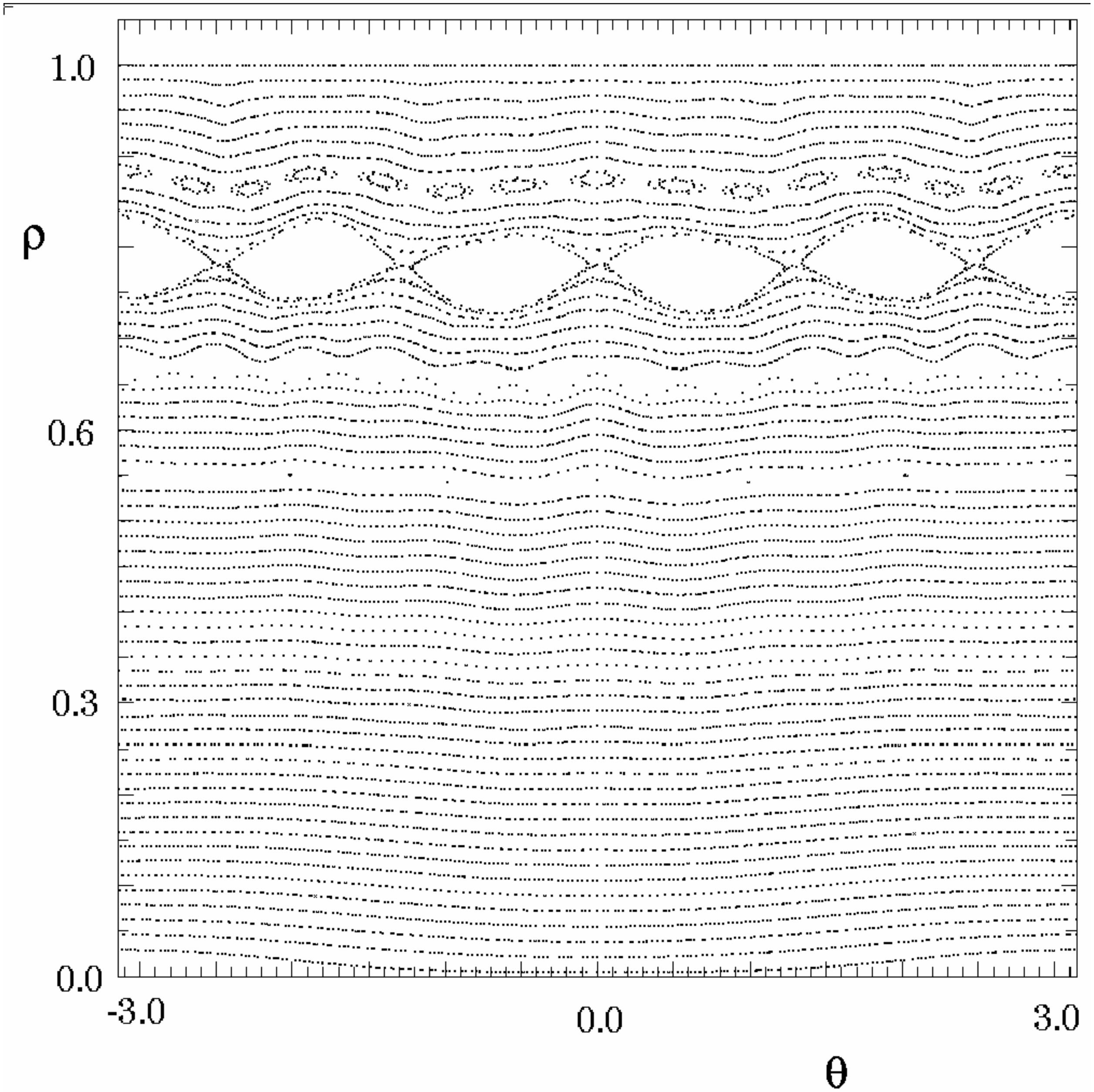


Early candidate configuration.

When β raised to 3%, PIES finds substantial flux surface loss.

Not a converged equilibrium. With pressure profile flattened in stochastic region, surfaces continue to deteriorate as calculation progresses, so that further computation is of limited interest.

(Note that, in addition to flattening pressure profile in stochastic region, calculation does not include potentially stabilizing neoclassical effects.)



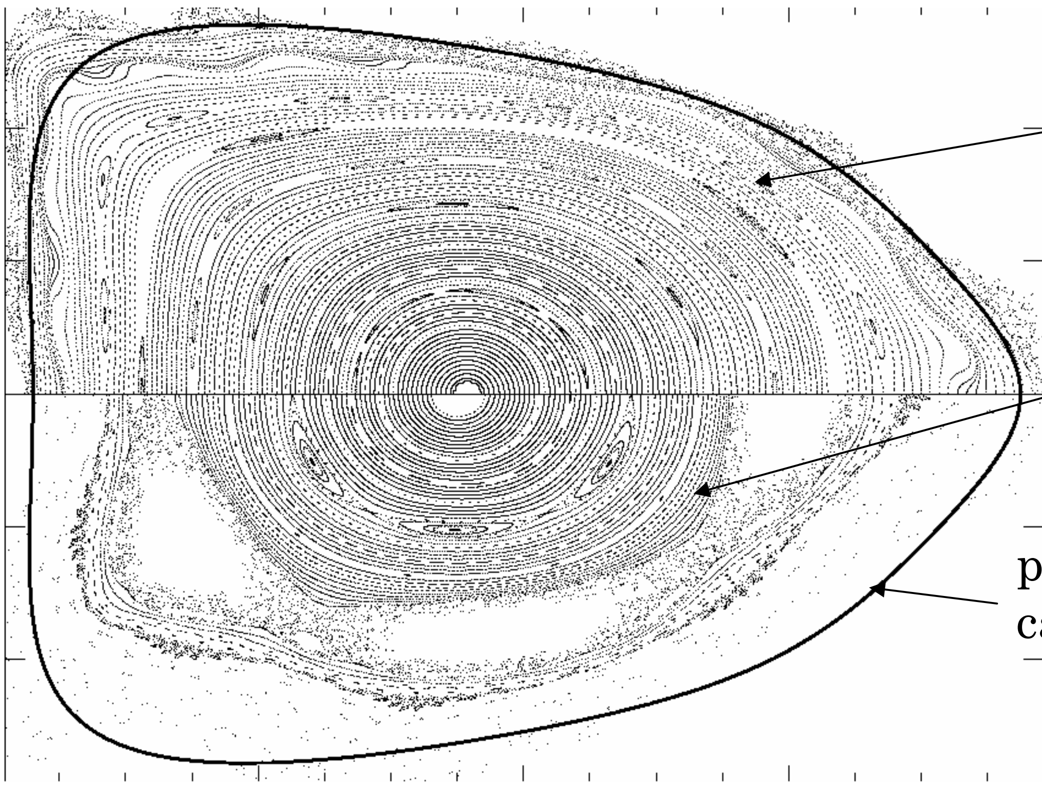
Reference Configuration

PIES calculation for reference configuration, as originally generated by optimizer, at full current, β = 4.2%. Flux surfaces intrinsically better than those of earlier configuration.

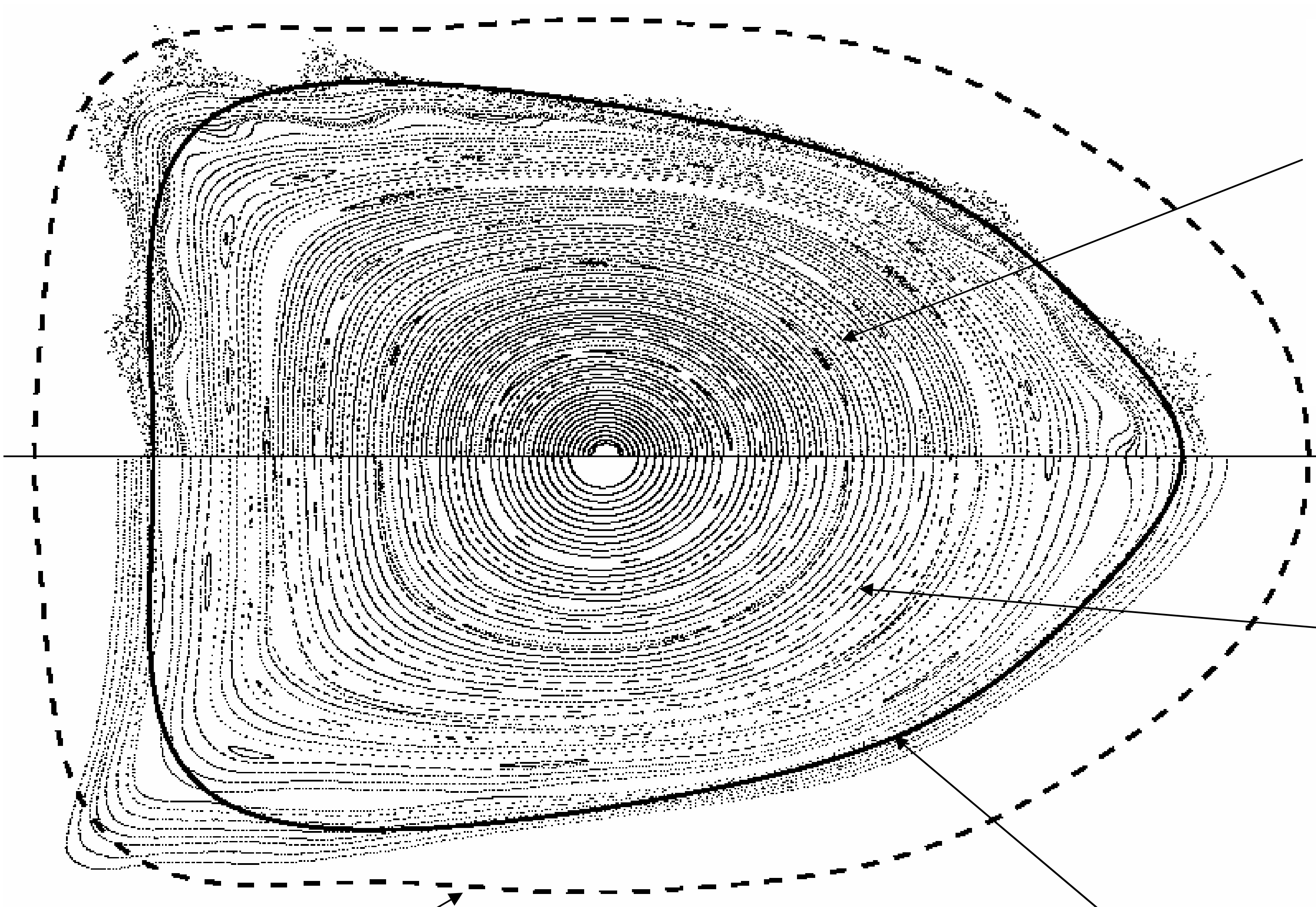
Width of island chain at ρ ≈ 0.8 can be greatly reduced by adjusting amplitude of corresponding resonant Fourier mode in specification of boundary shape.

PIES Calculations for NCSX: 2. Design of Coils to Preserve Flux Surfaces

An optimizer has been built around the PIES code to modify the coil design to heal islands while preserving desired physics and engineering properties.



Multi-Filament Model Improves Flux Surfaces



Converged free-boundary PIES calculation using single-filament model for coils.

Calculation with multi-filament model.

first wall

plasma boundary in VMEC calculation with unhealed coils.

Summary and Conclusions

- PIES equilibrium calculations for W7AS performed using reconstructed profiles from VMEC-based optimizer.
- Calculated equilibria have edge stochastic region above threshold value of β. Width of stochastic region increases with increasing β.
    - o Consistent with conventional wisdom that compression and distortion of 3D surfaces due to Shafranov shift breaks surfaces.
    - o Also consistent with analytical model for this effect (Reiman and Boozer, Phys. Fluids **27** (1984)).
- Calculated width of stochastic region sensitive to pressure gradient in stochastic region.
- Coil designed to control resonant magnetic perturbations near plasma edge has strong effect on width of stochastic region.

- Field line trajectories in stochastic region behave as if flux surfaces punctured near outer midplane but remain intact elsewhere.
- Calculated magnetic diffusion coefficient for the two shots consistent with observed differences in attainable β, and consistent with differences in reconstructed pressure profiles.
- Estimate of resulting energy transport consistent with expectation that contribution of stochasticity to transport substantial, but not sufficiently large to suppress pressure gradient.
- PIES calculations predict that both W7X[1] and NCSX will have improved robustness of flux surfaces at high β.

[1]Calculations by Merkel and Drevlak.