MHD Activity Induced Symmetric Breaking in Tokamaks and Its Consequences

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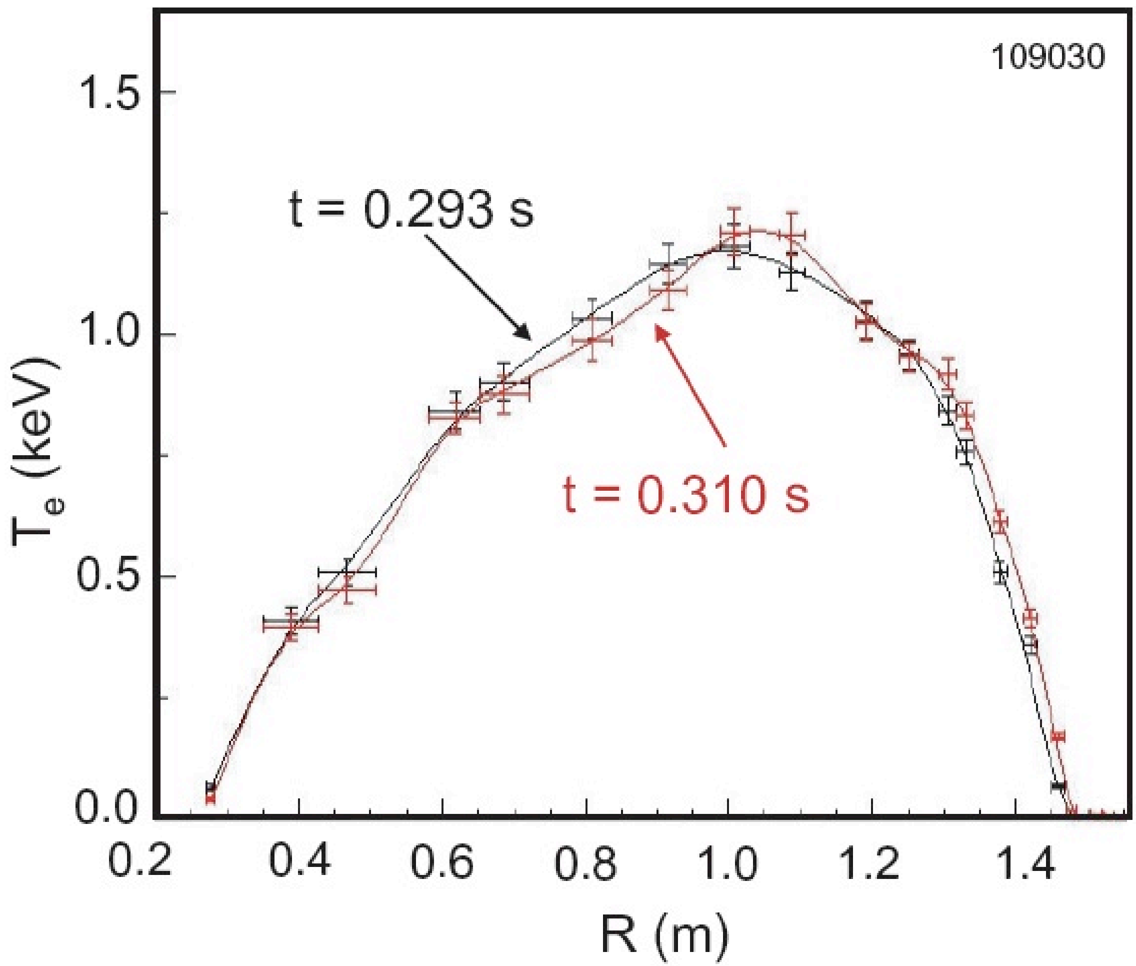
Outlines

- **MHD activity distorts the magnetic surface and breaks toroidal symmetry in |*B*| in tokamaks.**
- **The asymmetric |*B*| enhances plasma viscosity.**
- **Enhanced toroidal plasma viscosity damps toroidal flow.**
- **Improved plasma confinement in the vicinity of an island (or low order rational surfaces).**
- **Island rotation frequency caused by the enhanced toroidal plasma viscosity.**
- **Island induced bootstrap current on the island evolution.**
- **Parallel heat flow in the vicinity of an island.**
- **Conclusions**

Distorted Electron Temperature Profile in NSTX*



- **When MHD modes appear, electron temperature profile is distorted. This is an indication that the magnetic surface is distorted.**

*** S. E. Sabbagh, *et al.*, Nucl. Fusion 44 , 560 (2004), and NSTX Result Review and Forum 2001.**

Magnetic Surface Distortion

- **A perturbed magnetic field $\boldsymbol{B}_1$ modify $|\boldsymbol{B}|$ directly [Smolyakov, PoP 1995]**

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1.$$

- **However, $\boldsymbol{B}_1$ also perturbs the magnetic surface, and produces another modification on $|\boldsymbol{B}|$ on the distorted magnetic surface [Shaing, PoP 2003].**

- **In Hamada coordinates, the equilibrium magnetic field is**

$$\boldsymbol{B}_0 = \bar{\psi}'\nabla\bar{V}\times\nabla\bar{\theta} - \bar{\chi}'\nabla\bar{V}\times\nabla\bar{\varsigma},$$

where $\bar{V}$ is the volume enclosed by the magnetic surface, $\bar{\theta}$ is the poloidal angle, $\bar{\varsigma}$ is the toroidal angle, $\bar{\psi}' = \boldsymbol{B}_0 \bullet\nabla\bar{\varsigma}$, $\bar{\chi}' = \boldsymbol{B}_0 \bullet\nabla\bar{\theta}$, and $\nabla\bar{V}\times\nabla\bar{\theta} \bullet\nabla\bar{\varsigma} = 1$.

- **The perturbed magnetic field can be expressed as**

$$\boldsymbol{B}_1 = \nabla\times(\xi\times \boldsymbol{B}_0),$$

where ξ the plasma displacement.

- **The displacement vector ξ can be decomposed into**

$$\boldsymbol{\xi} = \xi^V \nabla\bar{\theta}\times\nabla\bar{\varsigma} + \xi^\theta \nabla\bar{\varsigma}\times\nabla\bar{V} + \xi^\zeta \nabla\bar{V}\times\nabla\bar{\theta},$$

where ξ^V, ξ^θ, and ξ^ζ are the contravariant components of ξ.

- **Combining $\boldsymbol{B}_0$ and $\boldsymbol{B}_1$, and keeping the linear terms,**

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1 = \psi'\nabla V\times\nabla\theta - \chi'\nabla V\times\nabla\zeta,$$

where, $V = \bar{V} - \xi^V$, $\theta = \bar{\theta} - \xi^\theta$, $\zeta = \bar{\varsigma} - \xi^\zeta$, $\psi' \equiv \bar{\psi}' - \bar{\psi}''\xi^V \equiv \psi'(V)$, and $\chi' \equiv \bar{\chi}' - \bar{\chi}''\xi^V \equiv \chi'(V)$.

- **The inverse Jacobian is $\nabla V\times\nabla\theta \bullet \nabla\zeta = 1 - \nabla\bullet\boldsymbol{\xi}$.**
- **If ξ is incompressible, *i.e.*, $\nabla\bullet\boldsymbol{\xi} = 0$, the new perturbed magnetic coordinates are also Hamada.**

- **On the perturbed magnetic surface,**

$$B = B_0(V, \theta, \zeta) + \xi^V (\partial B/\partial V) + \xi^\theta (\partial B/\partial\theta) + [\xi^\zeta (\partial B/\partial\zeta)].$$

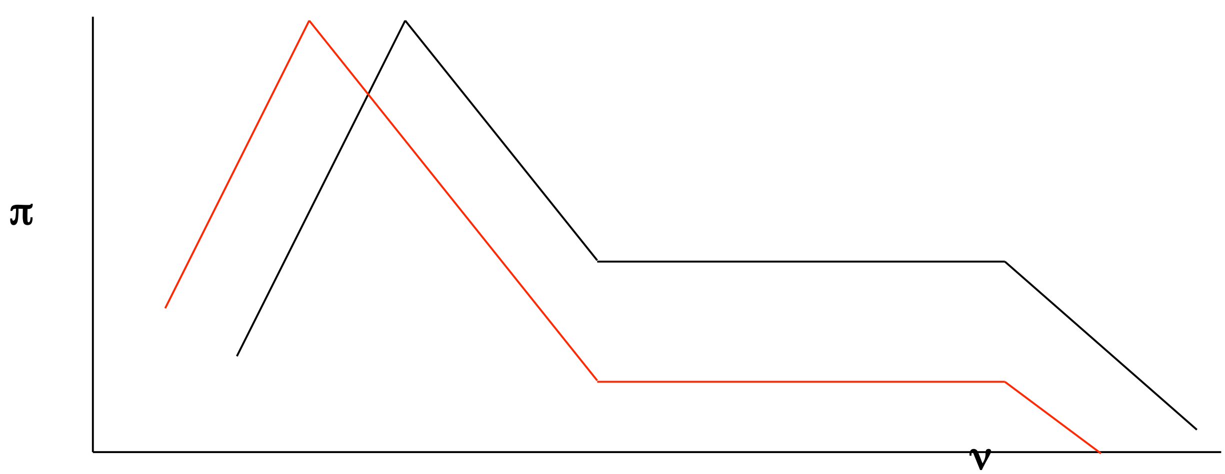
- **Thus, besides the direct change in $|B|$, the surface distortion produces another change in $|B|$ on the perturbed magnetic surface.**

- **The perturbed |*B*| is no longer toroidally symmetric just like stellarators.**

- **Stellarator physics becomes relevant to tokamak confinement.**

- **However, the more relevant physics is not from the helically trapped particle orbits but from the wobbling banana orbits under the influence of the helically perturbed magnetic field.**

- **Typical collision frequency dependence of the toroidal viscous force**



Electrons: ________
Ions: ________

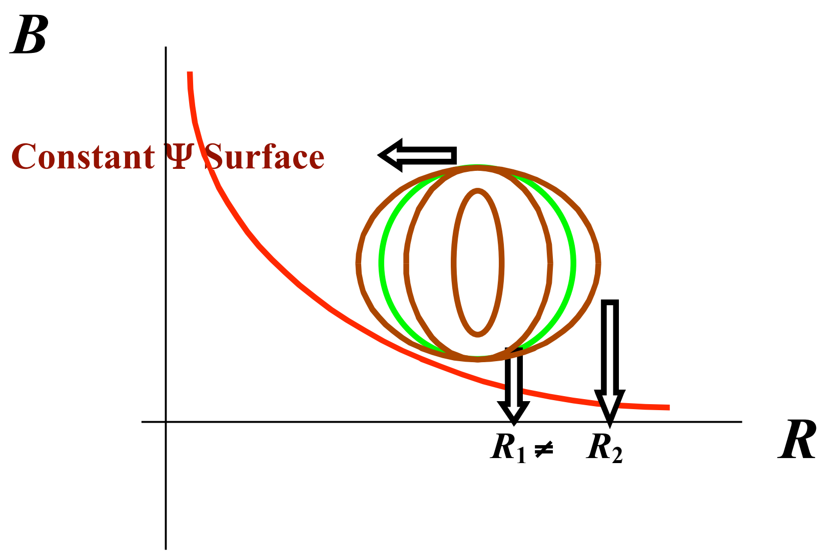
- **Only results in the 1/ν regime will be presented here although results in all other regimes have been calculated.**

- **The onset of the 1//ν regime is $\nu_* < 1$.**

- **The reason that |*B*| on the perturbed magnetic surface is important is because that transport fluxes are calculated on the perturbed magnetic surface.**

<u>**Magnetic Surface Distortion in the Vicinity of an Island**</u>

- **When an island is imbedded in a tokamak, $|B|$ in the vicinity is distorted [Shaing, PRL 2001]:**



$$B/B_0 = 1 - \left[\frac{rs}{R} \pm \frac{rw}{R}\left(\overline{\Psi} + \cos\xi\right)^{1/2}\right]\cos\theta$$

$\overline{\Psi}$: Normalized helical flux function,
ξ : $m(\theta - \zeta / q_S) + \omega t$, helical angle,
m : Poloidal mode number,
ζ : Toroidal angle.
r_w : A measure of the width of the island. ($2\sqrt{2}\, r_w = w$)

- **Note that the frequency definition we use here is opposite to the conventional definition.**

Enhanced Plasma Viscosity Caused by Asymmetric $|B|$

- **It is well known that the toroidal flow damping rate of the standard neoclassical toroidal plasma viscosity is too slow to explain what observed in tokamak experiments.**

- **The broken toroidal symmetry in $|B|$ enhances toroidal plasma viscosity and increases toroidal flow damping rate.**

- **For example, toroidal plasma viscosity (or asymmetric particle fluxes) for the non-resonant MHD activity in the $1/\nu$ regime is:**

$$\Gamma_j = - N[(v_{tj})^4/(4\pi^{3/2})][(cM_j/e_j\chi')^2/\nu_j][\lambda_{1j}(p_j'/p_j + e_j\Phi'/T_j)$$

$$+ \lambda_{2j} T_j'/T_j] I_\lambda,$$

where λs are numerical numbers, and

$$I_\lambda = \int_{\lambda_m}^{\lambda_M} d\lambda\, \lambda\, [\oint d\theta\, (|v_{\|0}|/v)]^{-1} \sum_n n^2 \{[\oint d\theta\, (|v_{\|0}|/v)\, A_n]^2$$

$$+ [\oint d\theta\, (|v_{\|0}|/v)\, B_n]^2\}.$$

- **This viscosity scales like $\varepsilon^{3/2}(\delta B)^2/\nu$.**

- **The asymmetric particle flux shown here is related to toroidal plasma viscosity through the flux-force relation for stellarators:**

$$\Gamma = -(c/e\chi'\psi')\langle \mathbf{B}_{\mathrm{p}} \bullet \nabla \bullet \pi \rangle = (c/e\chi'\psi')\langle \mathbf{B}_{\mathrm{t}} \bullet \nabla \bullet \pi \rangle,$$

where $\mathbf{B}_{\mathrm{p}} = -\chi'\nabla V \times \nabla\zeta$, $\mathbf{B}_{\mathrm{t}} = \psi'\nabla V \times \nabla\theta$.

- **The thermodynamic force can also be cast in terms of flows:**

$$(p'/p + e\Phi'/T) = -(e/cT)(\chi' V^{\zeta} - \psi' V^{\theta}).$$

- **Thus, toroidal plasma viscosity depends on both the toroidal flow and the poloidal flow.**

- **In general, toroidal flow evolution is coupled to that of the poloidal flow in all regimes.**

- **The toroidal flow damping rate caused by this viscosity is observed in NSTX [Zhu, Sabbagh, Bell, *et al.*, PRL 2006].**

- **There is also a steady toroidal flow associated with this viscosity. In the 1/ν regime, it is:**

$$V_\zeta = 3.54 \frac{cT_i}{e_i B_p} \frac{dT_i/dr}{T_i},$$

if the poloidal flow is the standard neoclassical value in the banana regime:

$$V_\theta = 1.17 \frac{cT_i}{e_i B} \frac{dT_i/dr}{T_i}.$$

- **In the plateau regime, and the Pfirsch-Schluter regime, however, the steady state toroidal flow is quite small. [Shaing, PF 1986; PoP 2004]. In the large aspect limit, the parallel flow is approximately zero.**

- **In the vicinity of an island, the toroidal plasma viscosity is also enhanced. For example, in the 1/ν regime**

$$\Gamma = -\frac{C_1}{2}\frac{\left(I\mathbf{n}_O\bullet\nabla\theta\right)^2}{M^{7/2}\Omega^2}\left(\frac{q_s'}{q_s}r_w\right)^2 m^2\delta_W^2\,\varepsilon_s^{3/2}\,\frac{F\left(\bar{\Psi}\right)\sqrt{1+\bar{\Psi}}}{\mathbf{K}\left(\kappa_f\right)}\times$$

$$\int dW W^{5/2}\frac{1}{\nu}\frac{\partial f_M}{\partial\Psi},$$

which scales like δ*B* instead of (δ*B*)^2. Thus, plasma viscosity force is further enhanced in the vicinity of an island.

- **If the island does not rotate, the radial electric field and the steady state toroidal flow speed have the same form in the asymptotic limits as those in the non-resonant perturbation cases except that the radial coordinate is replaced by the helical flux function.**

- **However, some islands do rotate. Symmetry breaking induced island plasma viscosity provides a mechanism to determine the island rotation frequency.**

Plasma Confinement in the Vicinity of an Island

- **In the vicinity of an island, the radial electric field has a radial scale length of the order of the island width.**

- **For example in the 1/ν regime,**

$$e_i\Phi'/T_i = -(p_i'/p_i) - 2.37\, T_i'/T_i\,.$$

- **The gradients here are taken at the helical flux surface.**

- **Invoking turbulence suppression theory [Shaing, *et al.*, IAEA 1988, PoP 1990], the turbulence in the vicinity of the island is suppressed.**

- **Thus, plasma confinement is improved.**

- **This provides an explanation as to why plasma confinement is improved in the vicinity of the low order rational surfaces observed in tokamak and stellarator experiments.**

- **Because turbulence is suppressed in the vicinity of an island, anomalous plasma viscosity is probably not important in that region. This makes the test of the theory easier in the experiments.**

- **A similar theory is also developed to explain extremely good plasma confinement in snakes [Shaing, PoP 2005].**

- **A snake is a helical structure with *m*=*n*=1 that is fueled by pellets [Weller, *et al.*, PRL 1987].**

Island Rotation Frequency

- **Island rotation frequency is important to the stability of the island through the polarization term [~ ω (ω - ω_{*i})] in the Rutherford equation [Smolyakov, PPCF 1993; Connor, *et al.*, PoP 2001].**

- **Island rotation frequency is determined by the sine component of the Ampere's law [Scott and Hasam, PF 1987; Jesen and Chu, Plasma Phys.1983]:**

$$\oint d\xi \int_{-\infty}^{\infty} dx \boldsymbol{J}_{\parallel} \boldsymbol{sin\xi} = \boldsymbol{c}\, \tilde{\psi} \Delta_s' /(4\pi \boldsymbol{R}).$$

- **The parallel current density has to have the right parities to contribute.**

- **The island induced plasma viscosity provides a mechanism to produce a parallel current density that has the right parities to determine the island rotation frequency.**

- **The island induced toroidal plasma viscosity drives a radial current density.**

- **From ∇•*J* = 0, there is a parallel current density.**

- **This parallel current density has the proper parities to contribute to the sine component of the Ampere's law.**

- **The island rotation frequency depends on the plasma collisionality because island induced plasma viscosity depends on the plasma collisionality.**

- **For example in the 1/ν regime, island rotation frequency is determined from the equation [Shaing, PoP 2002, and 2003]:**

$$-(2\pi)^{-5/2} (v_{ti}/Rq)^2 (r_W/\nu)\varepsilon^{3/2} m^2 (\delta_w^2/4) \times$$

$$I_{1/\nu} C_{1/\nu} \{\lambda_{T1i} [\omega_{*pi} + (\omega - \omega_{E0})] + \lambda_{T2i}\, \omega_{*Ti}\}$$

$$= n\varepsilon^2 (V_{AP}^2/4\pi)(S^2/4)(r_W/r_S)^4 \Delta_s',$$

where

$C_{1/\nu} \approx \pi$, $\omega_{*pi} = (mcT/eq)(p_{0i}'/p_i)$,
$\omega_{*Ti} = (mcT/e_j q)(T_{0i}'/T_i)$,
$V_{AP} = B_p/(4\pi NM)^{1/2}$,
n : **toroidal mode number, and**
$S = r_S (dq/dr)/q$.

- **Similar equations for the other collision frequency regimes are also derived. The main differences are the λs except when the electron viscous force becomes dominant.**

- **In the cases where electron viscous force dominates, the ion quantities are replaced by the electron quantities.**

- **If the island does not interact with the wall, in the 1/ν regime,**

$$(\omega - \omega_{E0}) = -\omega_{*pi} - (\lambda_{T2i} / \lambda_{T1i})\omega_{*Ti} .$$

- **This result implies that island rotation frequency is in the direction of the ion diamagnetic flow. (Recall that the definition of rotation frequency is opposite to the conventional one.)**

- **Island rotation frequencies in the other regimes are also calculated. In the regimes where ion plasma viscosity dominates, island rotation frequency is in the direction of the ion diamagnetic flow if it does not interact with the wall.**

- **This conclusion is consistent with the numerical calculations using Braginskii equation [Sugiyama and Park, PoP 2000].**

- **In the regime where the electron plasma viscosity dominates, island rotation frequency reverses to the direction of the electron diamagnetic flow.**

- **Note that this island rotation theory does not have undetermined parameters and can be tested in experiments.**

- **Especially the collisionality dependence is unique to the theory.**

Island Induced Bootstrap Current on Island Evolution

- **Because the toroidal symmetry in |*B*| is broken in the vicinity of an island, the bootstrap current is not the same as that in a symmetric tokamak.**

- **There is an additional bootstrap current density driven by the broken symmetry.**

- **This additional island induced bootstrap current modifies the Rutherford equation [Shaing and Spong, PoP 2006]:**

$$\frac{\partial r_w}{\partial t} = 0.43\frac{\eta c^2}{4\pi}G(\varepsilon, \nu_{*e})[\Delta' - \frac{\beta_p}{\pi}\frac{q_s}{q_s'}\frac{\sqrt{\varepsilon}}{r_w}\frac{1}{p_{to}}\frac{\partial p_{to}}{\partial r}(8.1F$$

$$+ \frac{0.434}{\nu_{*e}}\frac{r_w^2}{r^2})],$$

where $\beta_p = 8\pi p_{to} / B_p^2$**, *F* and *G* are order of unity, and** p_{to} **is the total equilibrium plasma pressure.**

- **For simplicity, only neoclassical island term [Carrera, *et al.*, PF 1986; Callen *et al.*, IAEA 1986] and the island induced bootstrap current term are kept here.**

- **The island induced bootstrap current is narrow in the radial direction. Thus, it has no significant observational consequence.**

- **The island induced bootstrap current term is stabilizing.**

- **The saturated island width is**

$$\frac{r_w}{r} = \frac{r\Delta' + \sqrt{(r\Delta')^2 - 4ab}}{2a},$$

where

$$\boldsymbol{a} = \frac{0.434}{\nu_{*e}} \frac{\beta_p}{\pi} \frac{q_s}{q_s'} \frac{\sqrt{\varepsilon}}{r_w} \frac{1}{p_{to}} \frac{\partial p_{to}}{\partial r},$$

and

$$\boldsymbol{b} = \mathbf{8.1}\boldsymbol{F} \frac{\beta_p}{\pi} \frac{q_s}{q_s'} \frac{\sqrt{\varepsilon}}{r_w} \frac{1}{p_{to}} \frac{\partial p_{to}}{\partial r}$$

- **Not all modes with negative Δ' are unstable. Only modes with**

$$|\Delta'| > |\Delta'_c| = \frac{1.2}{\sqrt{\nu_{*e}}} \sqrt{F(\varepsilon, \nu_{*e})} \frac{q_s}{r q'_s} \sqrt{\varepsilon}\, \beta_p \left| \frac{dp_{t0}/dr}{p_{t0}} \right|$$

are unstable.

- **High β_p is favorable for the stabilization.**

- **This may provide an alternative route to stabilize islands in tokamaks by tailoring current density profile and increasing β_p.**

- **This may also provide an explanation as to why low *m* islands are not always observed in tokamaks [Fredrickson, PoP 2002].**

- **Note that typical value of ν_{*e} is about 10^{-2}.**

Parallel Heat Conduction in the Vicinity of an Island

- **It is important to know if the plasma gradients are flattened or not inside an island [Fitzpatrick, PoP 1995].**
- **One of the relevant quantities is the parallel heat flow that determines the temperature profile inside an island.**
- **It has been argued that parallel heat conduction is non-local in the sense that the collisional mean free path is longer than the nominal connection length of the helical structure of the island.**
- **However, self-consistent non-local transport theory, whether it is radial or parallel transport problems, is difficult because one has to solve kinetic equation with a nonlinear Coulomb collision operator.**
- **This is a difficult task even numerically. There is no such code available.**
- **Fortunately, when one takes into account orbit squeezing, radial transport problem becomes local even when the nominal orbit width is comparable to the radial scale length.**

- **This is because the real orbit width is less than the radial scale length due to the effects of the orbit squeezing.**

- **In the parallel direction, orbit squeezing is not applicable of course.**

- **However, the parallel temperature gradient that results either from the magnetic reconnection or from the linear instability is very mild.**

- **This effectively increases the real connection length and the parallel transport problem becomes local in the parameter regimes that are found in typical tokamaks.**

- **Drift kinetic equation for parallel heat transport:**

$$v_{\parallel} \boldsymbol{n} \bullet \nabla f + \frac{e}{M} v_{\parallel} E_{\parallel} \frac{\partial f}{\partial E} = C(f),$$

- **The parallel gradient** $\boldsymbol{n} \bullet \nabla$:

$$\boldsymbol{n} \bullet \nabla = (\boldsymbol{n} \bullet \nabla \theta) \frac{\partial}{\partial \theta} + (\boldsymbol{n} \bullet \nabla \theta) m \left(1 - \frac{q}{q_s}\right) \frac{\partial}{\partial \xi} .$$

- **Because the width of an island is narrow:**

$$(1 - q/q_s) \sim w/a \ll 1.$$

- **We solve drift kinetic equation using $w/a \ll 1$, as an expansion parameter.**

- **The leading order equation is**

$$v_\parallel \boldsymbol{n}\bullet\nabla\theta \frac{\partial f_0}{\partial \theta} = C(f_0).$$

- **The solution to the leading order equation is a Maxwellian distribution**

$$f_0 = f_M(\Psi, \xi).$$

- **The next order equation is**

$$v_\parallel \boldsymbol{n}\bullet\nabla\theta \frac{\partial f_1}{\partial \theta} + v_\parallel \boldsymbol{n}\bullet\nabla\theta\, m\, (1 - \frac{q}{q_s}) \frac{\partial f_M}{\partial \xi} + \frac{e}{M} v_\parallel E_\parallel \frac{\partial f_M}{\partial E} = C(f_1).$$

- **This is the equation to be solved for the local parallel transport fluxes.**

- **For the local transport theory to be valid, the ordering procedure we display here must hold.**

- **This requires that**

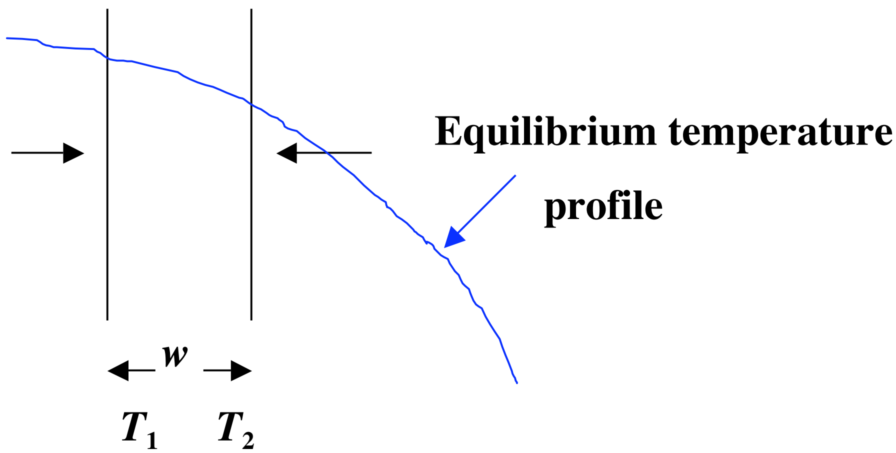
$$\nu > \mathrm{v}_t \boldsymbol{n} \bullet \nabla \theta \, \boldsymbol{m} \, (\mathbf{1} - \frac{q}{q_s}) \, \frac{1}{f_M} \frac{\partial f_M}{\partial \xi}.$$

- **If** $(\partial f_M / \partial \xi)/f_M \sim$ **1, the criterion for the local transport theory to be valid becomes**

$$\nu_* > \boldsymbol{m} \varepsilon^{-3/2} \frac{w}{a}.$$

- **For the typical value of the** $\nu_* \sim 10^{-2}$ **in the plasma core, and the typical island width** ***w*/*a*** **~ 0.01-0.1, the criterion is obviously violated.**

- **However, $(\partial f_M / \partial \xi)/ f_M$ is not of the order of unity when the variation of the plasma temperature is caused by the magnetic reconnection or by the linear instability.**

- 

$(\partial f_M / \partial \xi)/ f_M \sim w/a < 1.$

- **When we take into account the mild variation of the temperature along the field line, the criterion for the local transport theory to be valid becomes**

$$\nu_* > m \varepsilon^{-3/2} (\frac{w}{a})^2.$$

- **For typical value of $\nu_* \sim 0.01$, the transport processes are local for $w/a < 0.03$.**

- **This value of the island width is usually larger than the critical island width w_c [Fitzpatrick, PoP 1995].**

- **When $w < w_c$, the profiles are not flattened. When $w > w_c$, the profiles are flattened.**

- **Thus, we conclude that parallel transport processes in MHD problems are usually local for the parameters of interests.**

- **Local parallel transport fluxes for both electrons and ions in the banana regime have been calculated [Shaing, PoP 2006] and can be easily implemented for the modeling purposes.**

Conclusions

- **Toroidal symmetry in $|B|$ in tokamaks is broken when there are MHD modes present.**

- **There are two types of change in $|B|$: one is the direct change, and the other is due to the distorted magnetic surface.**

- **The toroidal plasma viscosity is enhanced and toroidal flow damping rate is increased.**

- **Radial electric field is also determined in the vicinity of an island.**

- **Because the radial electric field in the vicinity of an island has a radial scale of the order of the width of the island, turbulence is suppressed and plasma confinement is improved.**

- **Symmetry breaking induced plasma viscosity also provides a mechanism to determine island rotation frequency.**

- **When the ion plasma viscosity is dominant, island rotates in the direction of the ion diamagnetic flow. When the electron plasma viscosity dominates, the direction of the island rotation reverses.**

- **Island induced bootstrap current modifies Rutherford equation, and provides a stabilizing influence on the island saturation.**

- **This may provide an alternative route to stabilize the island by tailoring current density profile and increasing** β_p.

- **Because the variation of the temperature on the magnetic surface that is caused by the magnetic reconnection or the linear instability is very mild, parallel heat flux is local.**