Influence of pressure-gradient, shear on ballooning stability

- a semi-analytic expression determining the influence of pressure-gradient and average shear on ballooning stability is determined
- this equation provides the marginal stability diagrams

- Collaboration with C.C.Hegna
- thanks also to N.Pomphrey, A.Ware, R.Torasso, N.Nakajima

The ballooning equation takes the form...

depends on s^2

$$\left[\frac{\partial}{\partial\eta}P\frac{\partial}{\partial\eta}+Q\right]\xi=\lambda\sqrt{g}^{2}P\xi$$

$$P=\frac{B^2}{g^{\psi\psi}}-g^{\psi\psi}L^2 \quad , \quad Q=2p'\sqrt{g}\left(G+\iota I\right)\left(\kappa_n+L\kappa_g\right)$$

where L is the integrated local shear $L=\int_{\eta_k}^{\eta}s(\eta')d\eta'$

the local shear, and variations in the local shear caused by profile variations will play an important role

The pressure-gradient & shear are varied

first-order change in pressure and transform

$$p(\psi) = p^{(0)}(\psi) + \mu \; \delta p(y)$$

$$\iota(\psi) = \iota^{(0)}(\psi) + \mu \; \delta\iota(y)$$

$$\text{where } y = \frac{\psi - \psi_b}{\mu}$$

zero-order change in gradients

$$p' = p^{(0)}{}' + \cancel{\mu} \; \delta p' \, \cancel{\mu}^{-1}$$

$$\iota' = \iota^{(0)}{}' + \cancel{\mu} \; \delta\iota' \, \cancel{\mu}^{-1}$$

two free parameters $\delta p'$, $\delta\iota'$

The coordinate response & perturbed ballooning equation are determined

- the coordinates are varied to preserve MHD equilibrium

$$\mathbf{x}(\psi,\theta,\zeta) = \mathbf{x}^{(0)}(\psi,\theta,\zeta) + \mu\, \mathbf{x}^{(1)}(y,\theta,\zeta)$$

- It is the only the local shear which is affected to zero-order

$$s = s^{(0)} + \left(1 + D_\iota\right)\delta\iota' + D_p \delta p'$$

- The perturbed ballooning equation takes the form

$$\left[\frac{\partial}{\partial\eta}\left(P+\delta P\right)\frac{\partial}{\partial\eta} + \left(Q+\delta Q\right)\right]\left(\xi+\delta\xi\right) = \left(\lambda+\delta\lambda\right)\sqrt{g}^{\,2}\left(P+\delta P\right)\left(\xi+\delta\xi\right)$$

The coefficients are :

$$\delta P = P_{p'}\ \delta p' + P_{\iota'}\ \delta\iota' + P_{p'p'}\ (\delta p')^2 + P_{p'\iota'}\ \delta p'\delta\iota' + P_{\iota'\iota'}\ (\delta\iota')^2,$$

$$\delta Q = Q_{p'}\ \delta p' + Q_{\iota'}\ \delta\iota' + Q_{p'p'}\ (\delta p')^2 + Q_{p'\iota'}\ \delta p'\delta\iota' + Q_{\iota'\iota'}\ (\delta\iota')^2,$$

Eigenvalue perturbation theory is applicable

The perturbed eigenvalue / eigenfunction has the form :

$$\delta\lambda = \lambda_{p'}\ \delta p' + \lambda_{\iota'}\ \delta\iota' + \lambda_{p'p'}\ (\delta p')^2 + \lambda_{p'\iota'}\ \delta p'\delta\iota' + \lambda_{\iota'\iota'}\ (\delta\iota')^2 + \text{ h.o. } + \ldots$$

$$\delta\xi = \xi_{p'}\ \delta p' + \xi_{\iota'}\ \delta\iota' + \xi_{p'p'}\ (\delta p')^2 + \xi_{p'\iota'}\ \delta p'\delta\iota' + \xi_{\iota'\iota'}\ (\delta\iota')^2 + \text{ h.o. } + \ldots$$

expressions for 1st derivatives are obtained :

$$\lambda_{p'} = \frac{\int \xi\ [\partial_\eta P_{p'}\partial_\eta + Q_{p'} - \lambda R_{p'}]\ \xi d\eta}{\int \xi R \xi d\eta}\ ,\ \lambda_{\iota'} = \frac{\int \xi\ [\partial_\eta P_{\iota'}\partial_\eta + Q_{\iota'} - \lambda R_{\iota'}]\ \xi d\eta}{\int \xi R \xi d\eta}$$

the variation in eigenfunction is determined by an operator (matrix) inversion

$$[\partial_\eta P \partial_\eta + Q - \lambda R]\xi_{p'} = \lambda_{p'} R\xi - [\partial_\eta P_{p'}\partial_\eta + Q_{p'} - \lambda R_{p'}]\xi$$

and 2nd order (and 3rd, 4th, . . .) derivatives are similarly obtained

$$\lambda_{p'p'} = \ldots\ldots, \lambda_{p'\iota'} = \ldots\ldots, \lambda_{\iota'\iota'} = \ldots\ldots,$$

Theory determines . . . .
if increasing p′ is stabilizing or destabilizing;
if a second stable region is likely to exist

$$\lambda\ ,\ \frac{\partial\lambda}{\partial p'}\ ,\ \frac{\partial\lambda}{\partial\iota'}\ ,\ \frac{\partial^2\lambda}{\partial^2 p'}\ ,\ \frac{\partial^2\lambda}{\partial p'\partial\iota'}\ ,\ \frac{\partial^2\lambda}{\partial^2\iota'}\ ,\ ..\ ,\ ..\ ,\ ..$$

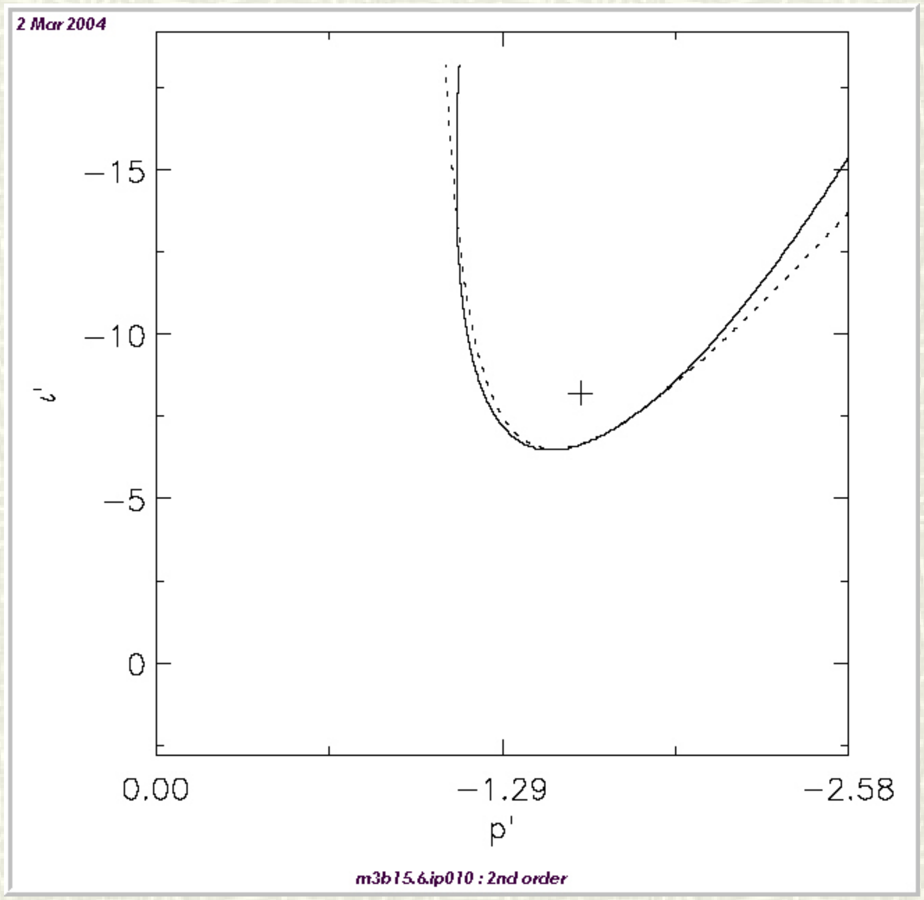
$\lambda < 0$ *stable*	$\lambda > 0$ *unstable*	$\lambda < 0$ *stable*
$\frac{\partial\lambda}{\partial p'} > 0$ 1st *stable*		$\frac{\partial\lambda}{\partial p'} < 0$ 2nd *stable*

if $\frac{\partial^2\lambda}{\partial p'^2} < 0$ then

a 2nd stable region is likely to exist

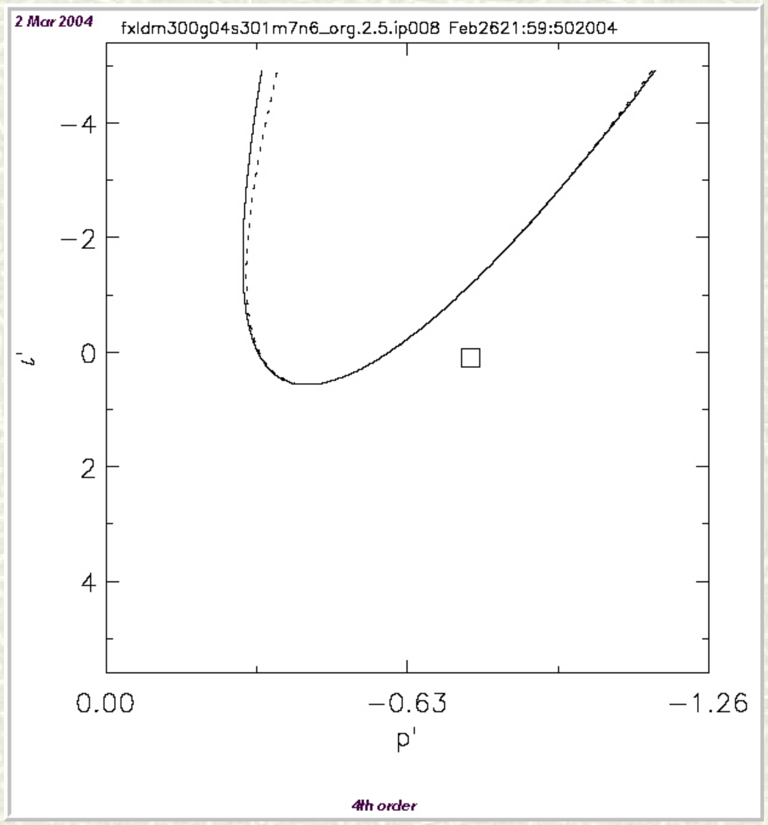
Quasi-poloidal (m3b15) configuration

- quasi-poloidal configuration studied by Ware et al. has strong second stable region
- solid curve is stability boundary determined by exactly re-solving ballooning equation on grid 200x200
- dotted curve from analytic expression – single eigenfunction calculation



LHD has second stable region near core

- LHD
- solid line is exact calculation; that is, solving the perturbed eigenvalue equation exactly on a grid 200x200
- dotted curve from analytic expression; requires only one ballooning calculation.



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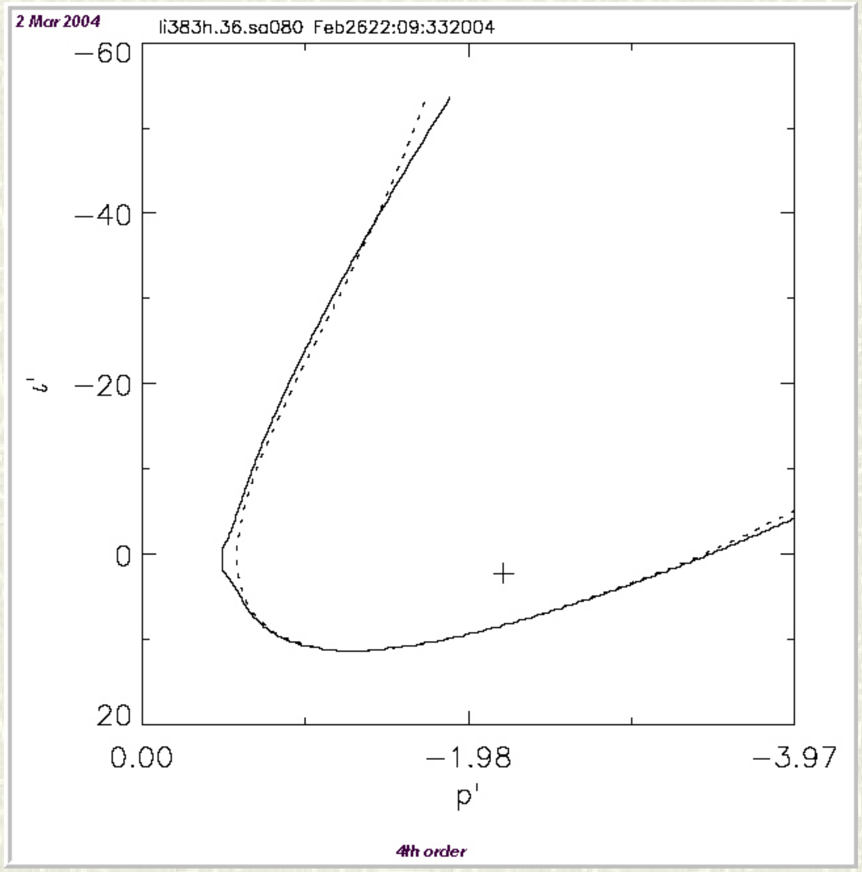

Usefulness of profile-variation method is verified by equilibrium reconstruction

A sequence of equilibria,
with increasing pressure,
is constructed.

Though the geometry is changing,
the marginal stability diagram
is a good predictor of stability limits

The equilibrium is indicated with
+ if unstable
– if stable