EQUILIBRIUM AND STABILITY IN STELLARATORS: THOUGHTS FROM A TOKAMAK PERSPECTIVE

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But the outrageous and just plain wrong statements are entirely my own!
MHD STABILITY CRITERIA APPEAR TO BE VIOLATED IN STELLARATOR EXPERIMENTS

- Stellarator experiments have substantially exceeded the stability limits predicted from local Mercier and ballooning code calculations:
  - LHD and W7-AS have significantly exceeded the predicted $\beta$ limits

- Global calculations are closer to experimental stability limits:
  - But even these appear to be violated in recent W7-AS experiments

- This superficially appears to be a quite different situation from the standard paradigm in tokamaks:
  - Ideal MHD appears to predict not just tokamak stability limits but also growth rates and mode structures in many situations

- How can the two situations be resolved?

⇒ Stellarators and tokamaks do have the same underlying physics based on Maxwell’s Equations and Newtonian mechanics!
AT SECOND GLANCE THE TWO SITUATIONS ARE NOT ALL THAT DIFFERENT

• Tokamaks also routinely violate some MHD stability limits:
  – MHD limits are open to interpretation and cannot be applied blindly as absolute hard limits
  – MHD limits can be sensitive to details in the equilibrium

• There are also some important distinctions between tokamaks and stellarators that may produce superficially different behavior
  – MHD theory, as applied to both, assumes the existence of nested flux surfaces:
    ⇒ In tokamaks this is sometimes not the case but normally it is an accurate assumption
    ⇒ In stellarators this is not always the case:

    Surfaces may not exist!
    They may exist but be non-nested!

  – We already know this to be partly true! But:
    ⇒ Given the sensitivity of the stability to the equilibrium the assumption of nested flux surfaces might be a poor approximation for stability even if islands are small
TOKAMAKS ALSO ROUTINELY VIOLATE SOME MHD STABILITY LIMITS

• The most well known example is the internal kink instability:
  – Tokamaks routinely operate with \( q < 1 \)
  – The sawtooth instability is a consequence of the internal kink but is not at all well described by it
    ⇒ Non-ideal effects are important for low growth rate modes
    ⇒ Nonlinear consequences are usually benign

• Tokamaks also routinely violate Mercier interchange stability limits:
  – The Mercier limit is normally close to the internal kink limit but appears to be largely irrelevant in tokamaks

• Ballooning modes can have consequences in tokamaks near ‘the \( \beta \) limit’:
  – Interchange modes are in principle a special case of ballooning
    ⇒ But the consequences of reaching the ballooning limit are not always devastating

• In H-mode Tokamaks also routinely reach intermediate \( n \) external mode stability limits:
  – ELMs appear to be the result of these instabilities
    ⇒ Nonlinear consequences are generally benign
IT IS NOT EVEN CLEAR THAT LOCAL MHD STABILITY CRITERIA SHOULD BE RELEVANT FOR STELLARATORS

- Localized modes predicted to be unstable for $\beta$ well below the global MHD limits should be stabilized by kinetic effects:
  - Finite $n$ corrections are needed for physically meaningful predictions
  - In tokamaks, finite toroidal mode number $n$ corrections to ballooning and Mercier stability are generally small
    $\Rightarrow$ The infinite $n$ calculation accurately reflects the real limit
  - In stellarators, the global stability codes in principle incorporate the high $n$ localized modes with low and intermediate $n$
    $\Rightarrow$ In practice the high $n$ modes are numerically excluded

- In tokamaks high and low $n$ are uncoupled and evaluated separately:
  - In Stellarators, they are coupled in principle and this is not accounted for in the localized criteria

- It is more realistic to ignore localized Mercier and ballooning limits in Stellarators and just use low and intermediate $n$ global calculations:
  - By excluding the high $n$ modes that in practice are stabilized by finite orbit effects the global codes are more closely reflecting the physics
    $\Rightarrow$ In the global calculations the range of $n$ needs to be terminated at the limit where finite orbit effects become important
TOKAMAK STABILITY LIMITS DEPEND SENSITIVELY ON THE EQUILIBRIUM

• It is not normally sufficient to fit the equilibrium to just the global characteristics of the discharge:
  − Stability depends quite sensitively on the details of both the current density (or safety factor) and pressure profiles
    ⇒ One can obtain widely varying results depending on the form assumed for the profiles for similar global parameters
    ⇒ Profiles need to be measured accurately and used in reconstructing the equilibrium for the stability calculations

• In Stellarators the equilibrium is believed to be known largely from the external coils: But
  − The $\phi$ profile is often taken from the vacuum profile
    ⇒ It is not normally measured in the discharge and may be different at finite $\beta$
  − The pressure profile is not known as a function of flux
    ⇒ At most it is measured as a function of space and the mapping to flux space needed for the equilibrium depends on the $\phi$ profile
THE ASSUMPTION OF NESTED FLUX SURFACES MAY NOT BE REASONABLE FOR ESTIMATING LINEAR STABILITY

- The assumption of nested flux surfaces may be invalid:
  - At least it may be an insufficiently good approximation to yield the observed stability
  - Finite $\beta$ can deteriorate the nested vacuum surfaces and given the sensitivity of the stability to the equilibrium configuration
    ⇒ Stability predictions using nested surfaces could be meaningless at finite $\beta$

- The islands and stochastic regions may be small but they may be ubiquitous throughout significant regions of the cross section:
  - Local flattening of the profiles and non-nested topology may yield very different stability from the ‘nearby’ nested configuration
    ⇒ The nested configuration may be linearly unstable but evolve nonlinearly to a configuration with ‘braided’ surfaces or thin islands, with flattened profiles in these regions
  - The new configuration will be linearly stable
    ⇒ The linear stability calculation using the approximate nearby nested flux surface equilibrium will yield the wrong result!
EQUILIBRIUM STABILITY AND TRANSPORT ARE NOT SEPARABLE IN STELLARATORS

• Existence of a nested flux surface equilibrium can be considered as either an equilibrium or a stability problem:
  – Unstable equilibria with nested surfaces will evolve to a nearby state with non-nested surfaces with lower energy if it is physically possible
  – Transport is strongly dependent on underlying equilibrium magnetic topology and in turn determines the possible equilibrium profiles

• Equilibrium codes can be considered stability codes:
  – An equilibrium computed under certain constraints must be stable unless those constraints can be avoided by a physically valid motion:
  – Otherwise any iterations for force balance in which an iterative error mimics an allowed perturbation will evolve away from the equilibrium unless constrained to not do so
  – A variational code will find the energy minimizing state unless constrained to not do so
VMEC, PIES, AND HINST EQUILIBRIUM CODES CAN GUARANTEE VARYING DEGREES OF STABILITY

• VMEC imposes simply nested flux surfaces:
  – Profiles assumed for $p(\psi)$ and a function specifying current density $j$
  $\Rightarrow$ Equilibria should be stable to all topology preserving and profile preserving (i.e. fixed $p(\psi)$ and $j$) MHD instabilities

• PIES and HINST have few constraints on the equilibrium:
  – Profiles assumed for $p(\psi)$ and a function specifying the current density $j$ (an integration constant on each flux contour for PIES)
  $\Rightarrow$ Equilibria should be stable to all profile preserving (i.e. fixed $p(\psi)$ and $j$) MHD instabilities
GUARANTEE OF STABILITY IS SUBJECT TO IMPORTANT CAVEATS

- Claim is that convergence to physically unstable equilibria is not possible unless constraints are imposed on the numerical procedure that prevent either:
  - Symmetry breaking perturbations away from force balance or
  - Equilibrium states without specific symmetries

  ⇒  Lack of convergence does not imply lack of stable equilibrium

- PIES and HINST assume $p = \text{constant}$ for flux surfaces inside islands:
  - Pressure is a different function of flux in separate simply connected regions
    ⇒  $p$ is not a single valued function of $\psi$
  - States with different prescriptions for the multiple values for $p$ and $j$ in different simply connected regions (islands etc.) are possible and may be physically accessible
    ⇒  The actual profiles will be determined by transport and the topology of the region
MAJOR QUESTION: SHOULD WE IGNORE MHD STABILITY PREDICTIONS BASED ON NESTED FLUX SURFACE EQUILIBRIAS

• Local stability criteria probably should be ignored:
  – There is little reason that infinite n should provide a physical limit
  – Finite n corrections appear to be large given the difference between the global code limits and the infinite n localized limits

• Global MHD stability is probably valid but must be applied to the right equilibrium:
  – Need to use the measured equilibrium profiles
  – May need to construct a non-nested flux surface equilibrium (with islands)
  – Flux surfaces might not even exist

• The nonlinear consequences are crucial in interpreting the results of a stability calculation:
  – Generally it might be expected that internal modes surrounded by a fairly robust and stable outer shell might be benign
  – Is there a way to quantify this without doing the full nonlinear calculation?
FINAL QUESTION: WHAT SHOULD WE DO? HOW SHOULD WE PROCEED?

• Is there a role for equilibrium and stability codes based on nested flux surfaces?
  – Under what conditions is nested surfaces a valid approximation for stability calculations?
  – Does linear instability of a nested flux surface equilibrium simply result in benign nonlinear evolution to a ‘nearby’ non-nested state?

• Can we formulate the stability problem in terms of finding nonlinearly stable equilibria:
  – Is it possible to develop a general equilibrium code with few imposed constraints that can guarantee stability
  – How can one distinguish a failure of the numerical scheme to converge from nonexistence of a stable nearby equilibrium?