Ballooning modes in quasi-symmetric devices: Effects of breaking symmetry

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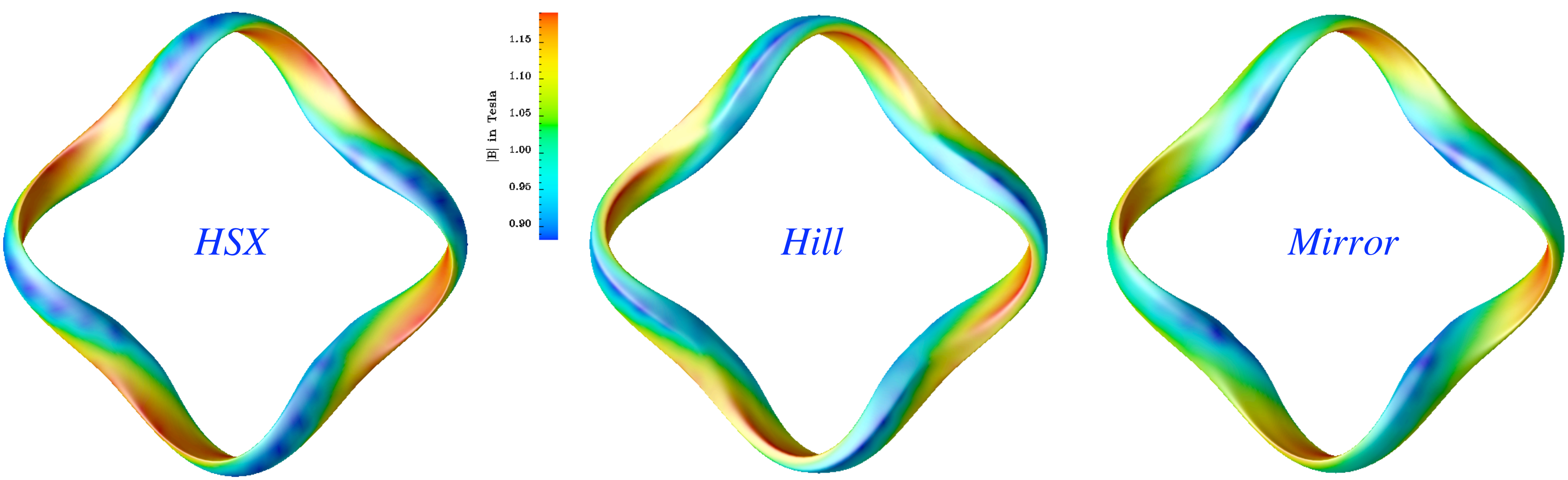
Overview

- Breaking symmetry:
  - For HSX, 3 different equilibria
    - *Standard, quasi-helically symmetric case*
    - *Mirror case*
    - *Hill case*
  - For QPS use a range of currents in the *TF* coils to modify the quasi-poloidal symmetry
- Examine the impact of symmetry breaking on ballooning stability
  - Effect on local shear and curvature
  - Ballooning results from COBRAVMEC

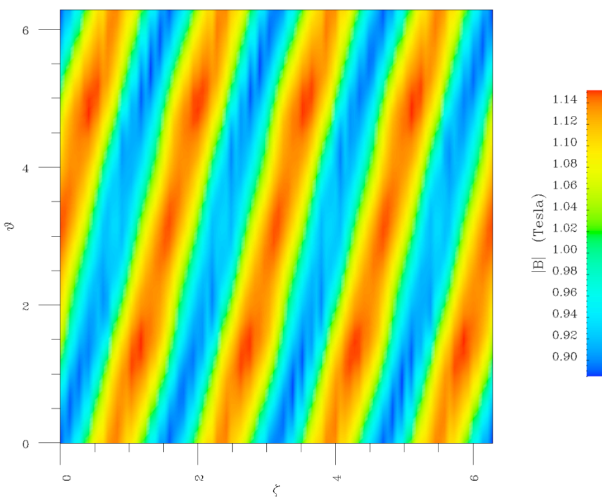
HSX has tested configurations that break its designed quasi-symmetry

- We have examined three different HSX cases:
  - ➢ *HSX*: standard, quasi-helically symmetric case
  - ➢ *Mirror*: adds a mirror term to break the symmetry
  - ➢ *Hill*: adds a hill to negatively impact stability

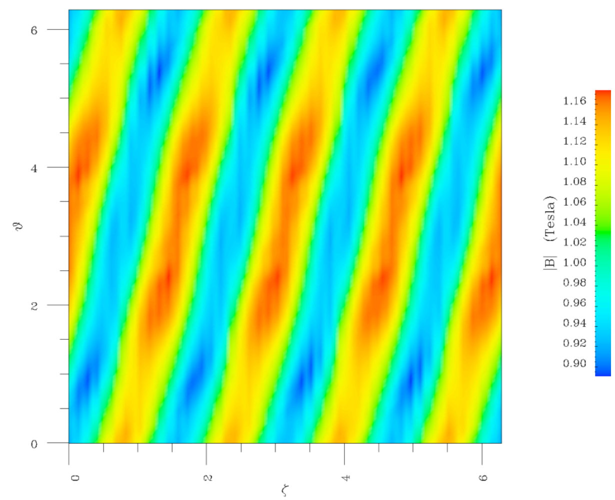


Both the *Hill* and *Mirror* cases have reduced quasi-symmetry

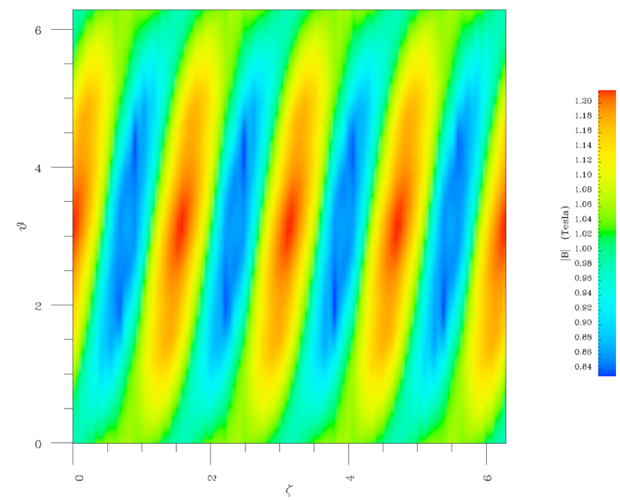
- Contour plots of $|B|$ in Boozer coordinates on the $S = 0.8182$ surface for all three cases
  - ➢ The reduced quasi-symmetry is noticeably visible



HSX



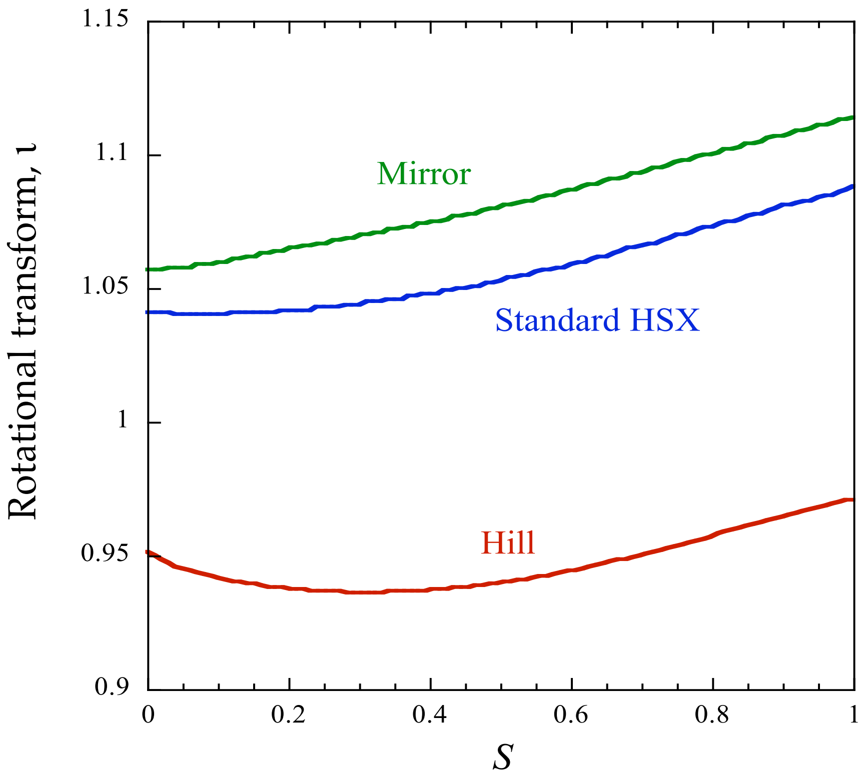
Hill

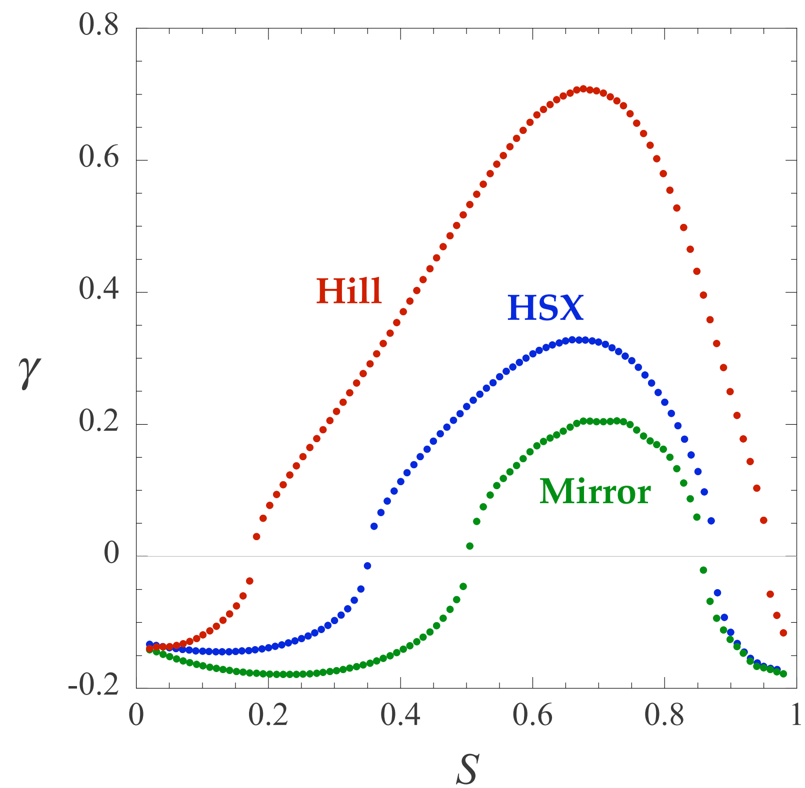


Mirror

The HSX cases have a wide range of equilibrium and stability properties

- Rotational transform profiles and ballooning growth rates ($\theta_k = 0$, $\alpha = 0$) at $\beta = 2.35\%$, $|B| = 1$ T, for each HSX case





Solving the ballooning eigenvalue equation in VMEC coordinates

- We use COBRAVMEC to solve the ideal MHD ballooning equation
  - Given a VMEC equilibrium (wout file), COBRAVMEC obtains the MHD eigenvalue as a function of the normalized flux, S, the field line label, $\alpha = q\theta - \zeta$, and the ballooning parameter, $\theta_k = k_q / k_\alpha$.

$$\left(\vec{B}\cdot\nabla\right)\left[\frac{|\nabla\alpha|^2}{B^2}\left(\vec{B}\cdot\nabla\right)\right]F + \left(\frac{R_0}{a}\right)^2 \frac{\beta_0 p'}{\Psi'^2}\kappa_s F + \lambda\frac{|\nabla\alpha|^2}{B^2}F = 0$$

  - To obtain: $\lambda = \lambda(S, \alpha, \theta_k)$

Solving the ballooning eigenvalue equation in VMEC coordinates (cont.)

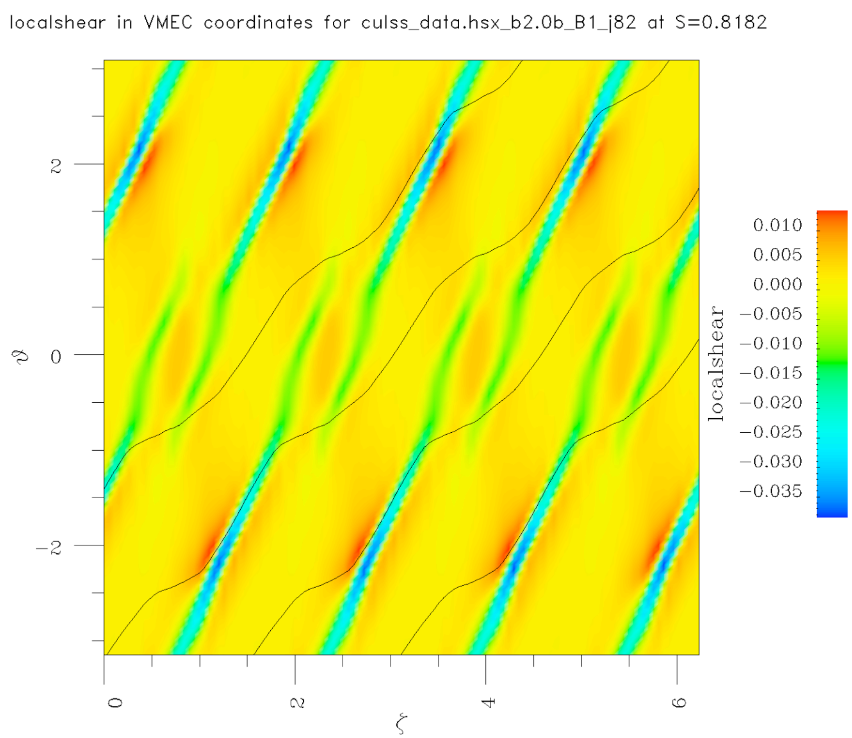
- The curvature that appears in the ballooning equation depends on both the VMEC normal and geodesic curvatures [see R. Sanchez, S. P. Hirshman, and H. V. Wong, *Computer Physics Communications* **135**, 82 (2001) for details]

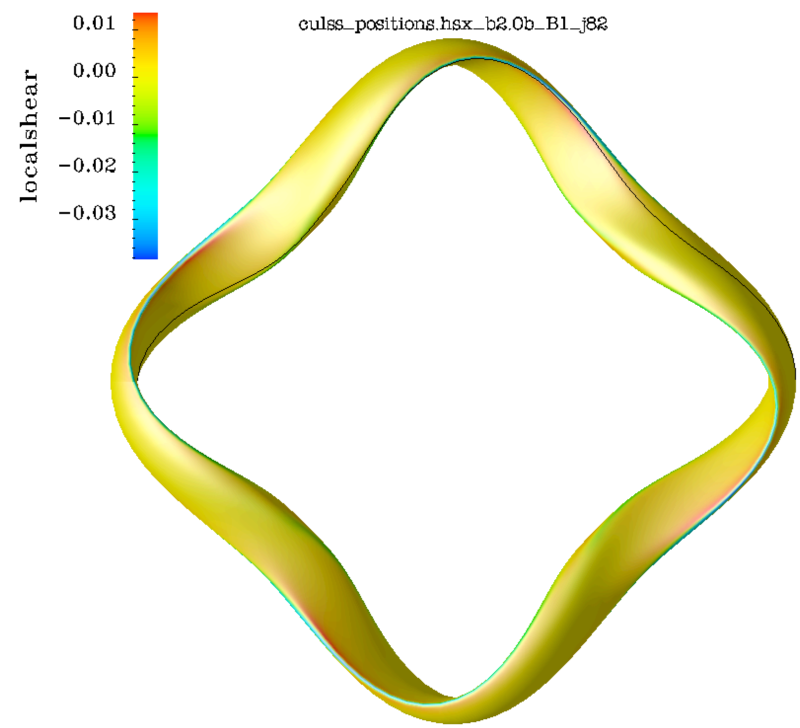
$$\kappa_s = \kappa_{s_V} + \kappa_{\theta_V} \left(\frac{\iota' \zeta_V - \partial \lambda_V / \partial s_V}{1 + \partial \lambda_V / \partial \theta_V} \right)$$

  - Where the terms with subscript V refer to VMEC coordinates and λ_V is the VMEC lambda
  - We will plot the normal curvature, κ_{sV}, and the geodesic curvature, $\kappa_{\theta V}$, in VMEC coordinates

The local shear in HSX is small except near the narrow portions of the configuration

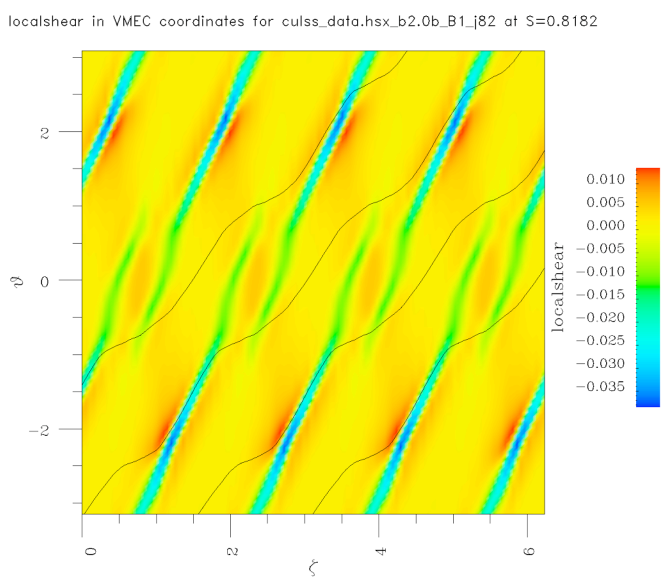
- Local shear for the standard HSX case on the S = 0.8182 surface, plane view and 3-D view



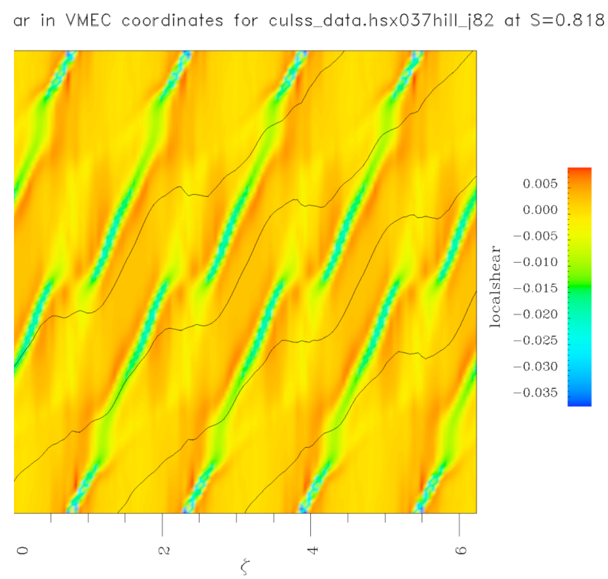


The local shear is similar in all three HSX configurations

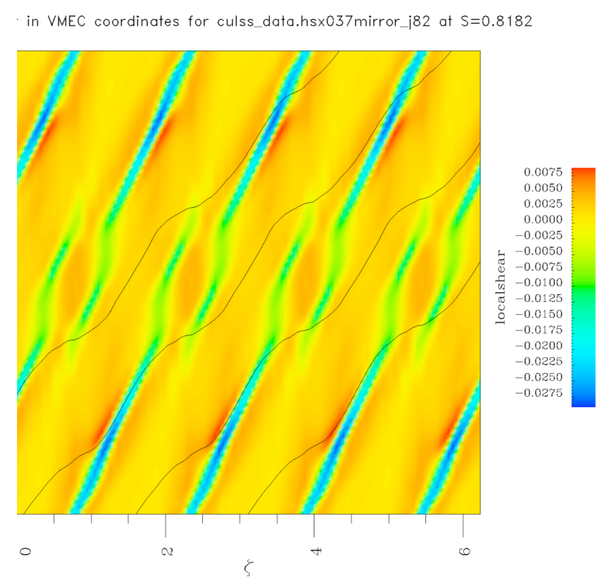
- Local shear on the $S = 0.8182$ surface, plane view, for the standard HSX, Hill and Mirror cases



HSX



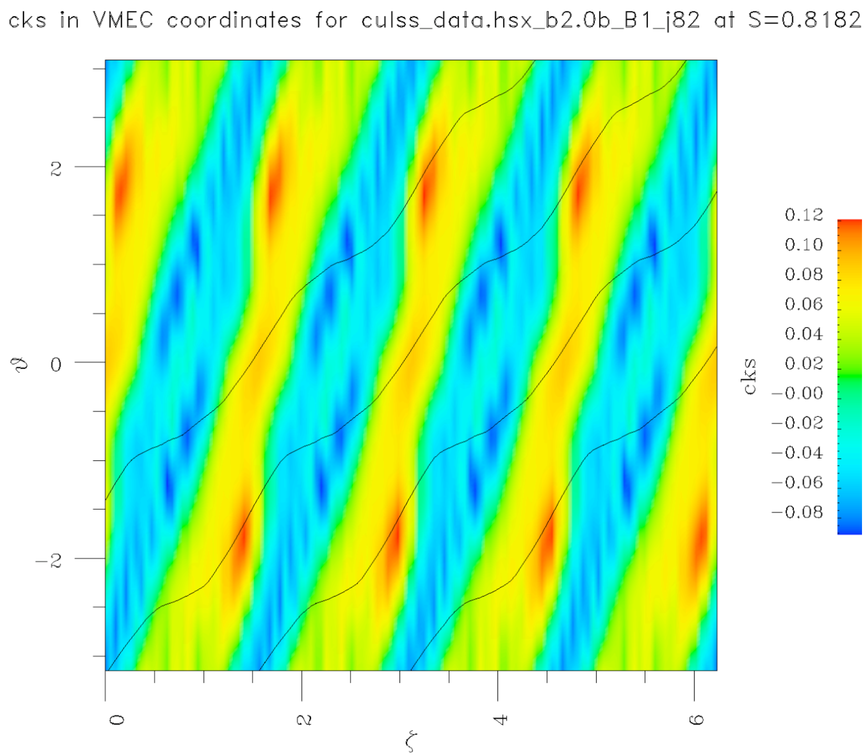
Hill

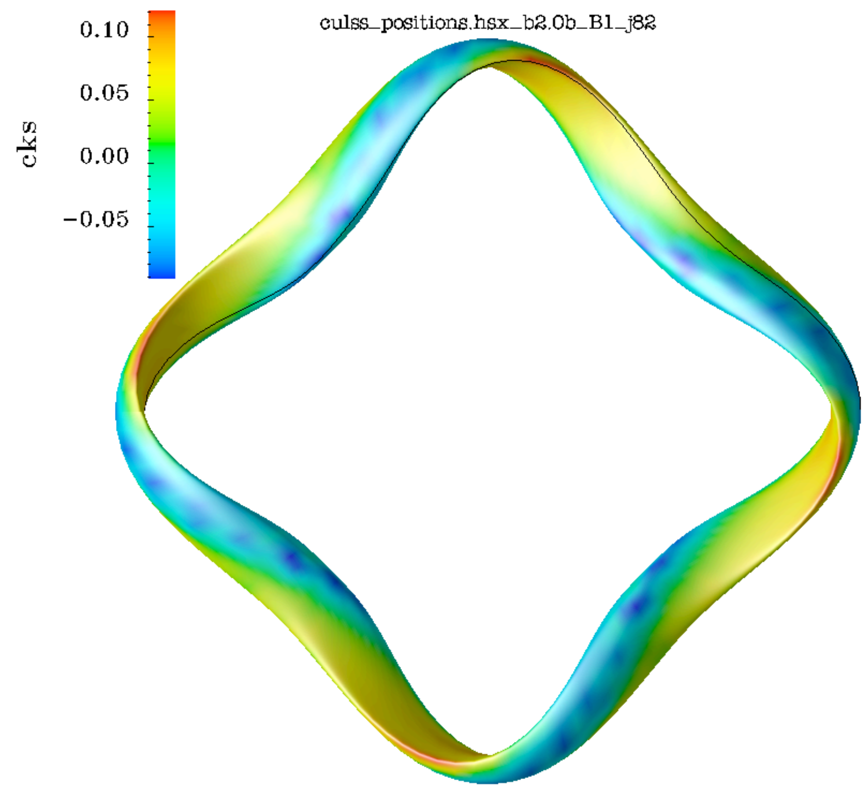


Mirror

Normal curvature in the standard HSX

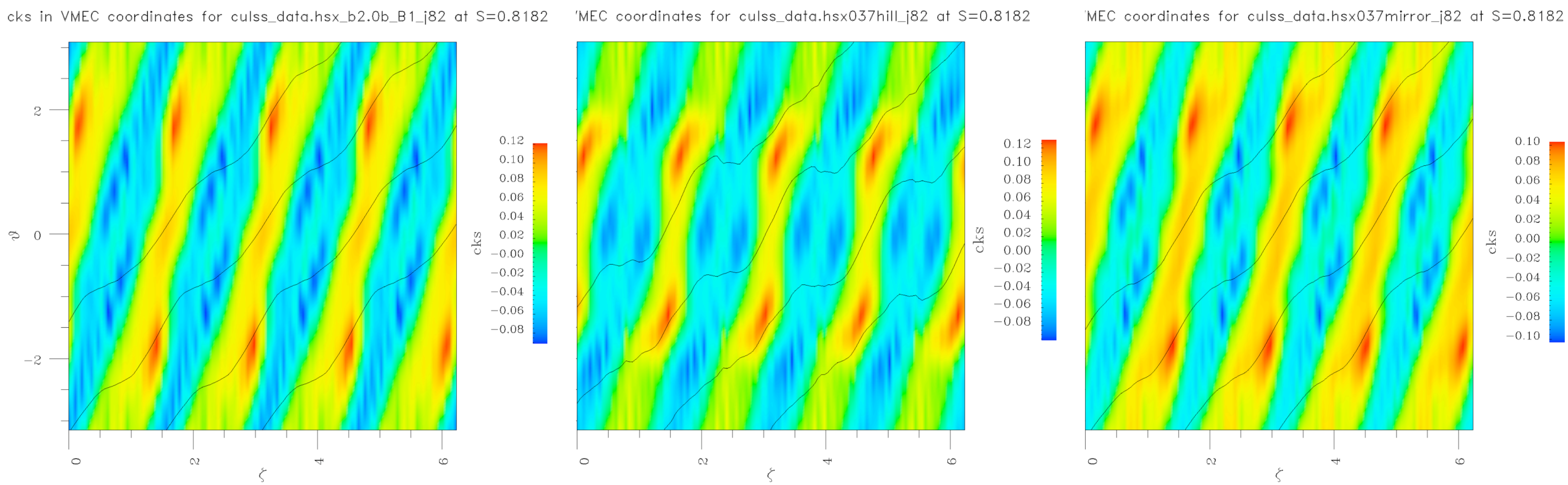
- Normal curvature for the standard HSX case on the S = 0.8182 surface, plane view and 3-D view





The normal curvature is different across the three configurations

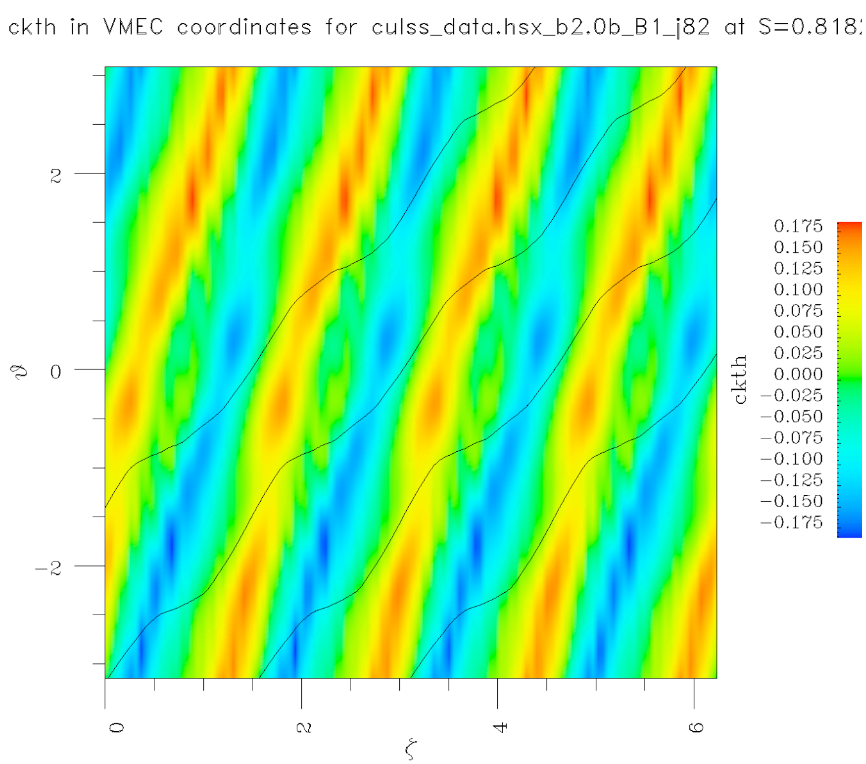
- Normal curvature on the S = 0.8182 surface, plane view, for the standard HSX, Hill and Mirror cases

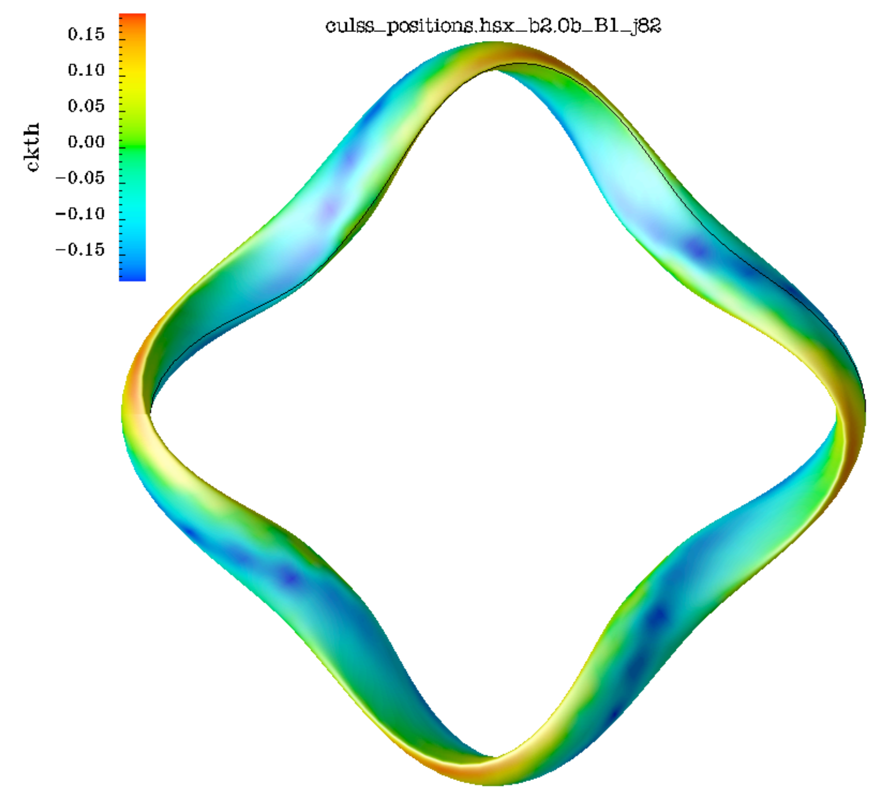


HSX *Hill* *Mirror*

Geodesic curvature in the standard HSX

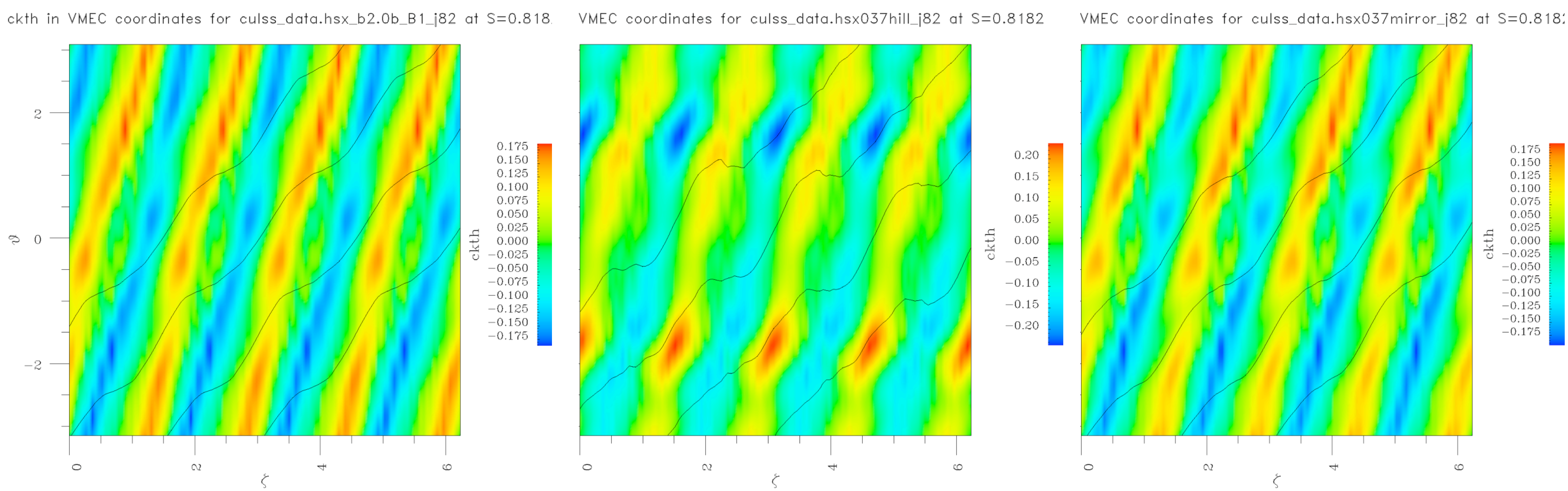
- Geodesic curvature for the standard HSX case on the *S* = 0.8182 surface, plane view and 3-D view





The geodesic curvature is very different for the *Hill* configuration

- Geodesic curvature on the S = 0.8182 surface, plane view, for the standard HSX, Hill and Mirror cases



HSX *Hill* *Mirror*

The Ray Equations

- Dewar and Glassers ray tracing theory can be used to obtain information about the global eigenmodes

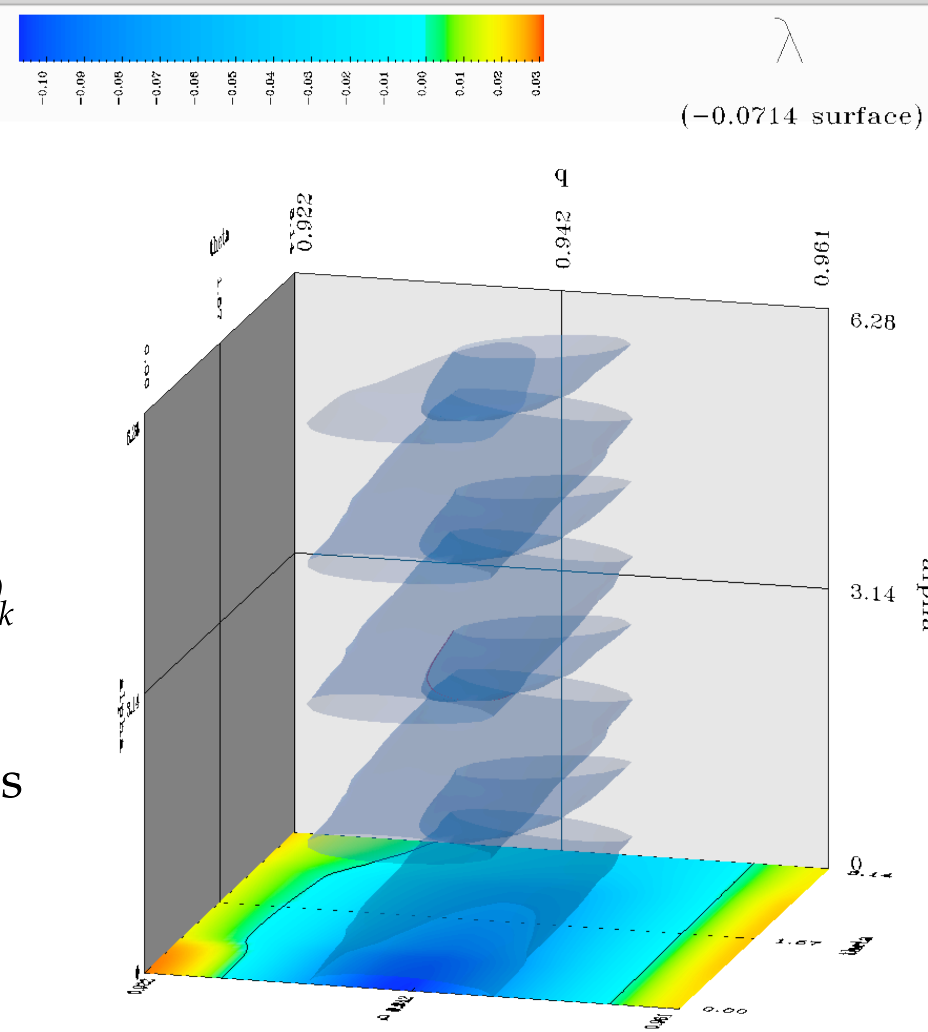
$$\lambda = \lambda(q, \alpha, \theta_k)$$

$$\dot{q} = \frac{\partial \lambda}{\partial k_q}, \quad \dot{\alpha} = \frac{\partial \lambda}{\partial k_\alpha}, \quad \dot{k}_q = -\frac{\partial \lambda}{\partial q}, \quad \dot{k}_\alpha = -\frac{\partial \lambda}{\partial \alpha}$$

- Using the values of λ on a 45x45x91 grid in (q, θ_k, α), we integrate the ray equations
  - Range for θ_k is 0 to π and for α is 0 to 2π

Ray tracing of ballooning modes in the standard HSX configuration

- Constant λ surfaces for the standard *HSX* case
  - $\beta = 2.35\%$
  - Surfaces of constant λ are localized in S, less localized in α, and extended in θ_k
  - The lack of dependence on θ_k makes tracing rays on these surfaces a challenge
  - Note: $S = 0.8182$, corresponds to the $q = 0.9302$ surface



Ray tracing of ballooning modes in the Hill configuration

- Constant λ surfaces for the *Hill* case
  - $\beta = 2.9\%$
  - Similar to the standard HSX
  - Stronger dependence on θ_k
  - Note: $S = 0.8182$, corresponds to the $q = 1.043$ surface