The effect on neoclassical transport of a fluctuating electrostatic (ES) spectrum

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We consider how transport changes in toroidal devices when one superposes on the background magnetic field $B$ a specified spectrum $\{\varphi_k\}$ of electrostatic (ES) modes, representing turbulence, or an externally-applied $E$-field.

- **2 intuitive pictures** for the effect:
  **(1) Additive** (superposition) picture:
  Commonly assumed that total diffusion coefficient $D$ is a sum of neoclassical and anomalous contributions,
  $$D = D_0^{nc} + D^{an},$$
  e.g., with $D^{an} \sim |\varphi|$ (strong turbulence), $D^{an} \sim |\varphi|^2$ (weak turbulence, quasilinear theory, some ripple transport).

  **(2) $\nu_{ef}$ picture**:
  One might instead expect the fluctuations to enhance the total effective collisionality $\nu_{ef} = \nu + \nu_{an}$ over the purely collisional rate $\nu$, shortening the decorrelation time.
Configurations:

- We study the transport in 3 configurations:
  (1) $7q_1\_tok$ = tokamak obtained from $7q_1$ by taking all Fourier components $B_{mn}$ of magnetic field strength $=0$ for $n\neq 0$.

- Plot $B(\theta, \zeta)$ and $B$(along field line):

(2) $7q_1$ = one variant of the quasiaxisymmetric stellarator LI383, on which NCSX is based.
$$j = \text{conventional (m,n)=(2,6) stellarator.}$$

-Ambipolar electric field $E_r$:

$$E_r = -\partial_r \phi_0, \quad \phi_0 = \alpha E (1 - \psi/\psi_a) \approx \alpha E (1 - r^2/a^2).$$
Perturbing Spectra:
- All configs have $q \in [2.53, 1.51] \approx [5/2, 3/2]$.
- Spectrum S1: Model turbulence with a small spectrum of low-n modes with $q_{mn} = m/n$ in this range: $m/n = \{3/2, 5/3, 2/1, 4/2, 6/3, 5/2\}$, with drift-wave (DW)-like frequencies,
\[\omega_{mn} = \alpha_\omega \omega_k \frac{1+k^2_\perp \rho^2}{(1+k^2_\perp \rho^2)},\]
amplitudes $e\varphi_{mn}/E_1 = \hat{\alpha}_m A_m(\psi)$, with $E_1 = 1$ keV,
\[\max(A_m(\psi)) = 1, \ \hat{\alpha}_m = 10^{-3} \alpha_A/(1+k^2_\perp \rho^2),\]
with $\alpha_\omega$, $\alpha_A$ multiplicative parameters, scanned in numerical studies.

Fig.4
**Spectrum S2**: As S1, but take all $n=0$.

- Spectrum S2 has larger $k_{||} \Rightarrow$ larger $E_{||} \Rightarrow$ enhanced capacity to break bounce-action $J_b$, energy $E$, and so enhance $\nu_{ef}$.

- S2 models externally-applied RF fields, such as employed on the Saturn stellarator[1] to detrap electrons[1]:


![Fig. 4. The ratio $r/r_o$ as a function of the central frequency $\tilde{f} = f_1 - f_2$, $\Delta f$ is the calculated range of reflection frequencies for the localized electrons found from Fig. 1.](image)

- More recently, some numerical studies have considered possible applications of externally-applied fields, detrapping electrons to control $E_r$,[2] entrapping ions for impurity removal[3].

Simulations:

- With background fields $\mathbf{B}(\mathbf{x})$, use GC code ORBIT to integrate the orbits of $N_p$ particles, taking a monoenergetic distribution of hydrogen ions with energy $E_0=1$ keV, launched halfway out $[r/a \equiv (\psi/\psi_a)^{1/2}]$ in a machine with major radius $R_0=1$ m, with $B_0 (=|\mathbf{B}|$ on axis of 3 Tesla.

- Compute diffusion coef $D$ from $D=\langle (\delta r_i)^2/2 \tau_i \rangle$, where $\langle F \rangle \equiv N_p^{-1} \Sigma_i F_i$ is an avg over all $N_p$ particles, $\delta r_i \equiv r_i - \langle r \rangle$, and $\tau_i$ is the run time for particle $i$, the smaller of its confinement time and a max run time $T$.
- Take $N_p=3000$, unless otherwise noted.
Take radial ambipolar field $E_r = 0$, & spectrum $S_1$:

- **Scan in collisionality $\nu$**:

![Graph](image.png)

- Banana $\rightarrow$ plateau regimes appear in $7q_1_{\text{tok}}$.

- $7q_1$ manifests modest $1/\nu$ regime, coalescing with $7q_1_{\text{tok}}$ curve at higher $n_{e0}$.

- $27j$ shows appreciable $1/\nu$ regime, as one expects for its much larger ripple.
-Scan in pert amplitude $\alpha_A$:
-Choose $n_{e_0} = 10^{13}/\text{cm}^3$, bit below onset of $1/\nu$ regime in Fig.5.

-Effect of $\alpha_A$ on tokamak consistent with both superposition and $\nu_{ef}$ pictures.
-Less effect on stellarator 27j on avg, consistent with $\nu_{ef}$ picture. Also, shows more structure than for tokamak.
-Subtracting off $\alpha_A=0$ contribution (from Fig.5):
-Scan in frequency ($\alpha_\omega$):

![Fig.8](image)

- Again subtracting off $\alpha_A=0$ contribution:

![Fig.9](image)

- **7q1_tok** has single central peak of halfwidth $\delta\alpha_\omega \approx 0.03$.
- **7q1** roughly follows 7q1_tok curve, plus additional structure at larger $\alpha_\omega$.
- **27j** manifests 2 significant features:
(a) The structure seen in 7ql is more pronounced in 27j, and shows a succession of peaks, with rough spacing $\Delta \alpha_\omega \approx 0.08$.
(b) For some $\alpha_\omega$, the DW spectrum can REDUCE $D[27j]$ below its $\alpha_\Lambda=0$ value.
Now, compare $E_r=0$ and $E_r\neq 0$, with spectra $S_1, S_2$. Focus on $27j$ henceforth:

- **Scan in $\nu$:**

![Graph showing log$_{10}(n_e0)$ vs. $\nu$ with curves for $\alpha_{E=0}$ and $\alpha_{E=.6}$](image)

$D[33\alpha,34\alpha] \nu$ dens

$\times 10^4$

$11.5$ $12$ $12.5$ $13$ $13.5$ $14$ $14.5$ $15$

$0$ $1$ $2$ $3$ $4$ $5$

$\log_{10}(n_e0)$
- Frequency scan \( (\alpha_\omega) \):

-Spectrum \( S2 \) produces larger effect than \( S1 \), as expected.

-For \( \alpha_E = 0 \) (puts ions in \( 1/\nu \) regime), see \( D^{an} < 0 \).
-For \( \alpha_E = 0.6 \) (puts ions in lower-\( \nu \) “superbanana regime”), see \( D^{an} > 0 \).

-Both results what expect for spectrum enhancing \( \nu_{ef} \).
Some Theory:

- Kinetic eqn: \( (\partial_t + L_H)f = Cf \),

\[ \text{(1)} \]

with Hamiltonian \( H(z) = H_0 + H_1 \), \( L_H = z^i \partial_i \),
\( z = \{ z^i \} (i=1-6) \) = parametrizing phase-space,
\( H_0 = \text{unperturbed } H \), given by background \( B(x) \),
and \( H_1 = \sum m e \phi_m \cos \eta_m \sim \alpha_A \), \( m = (m,n) \), \( \eta_m = n \zeta - m \theta - \omega_m t \).

- Neoclassical theory follows from (1) with \( H_1 \sim \alpha_A \to 0 \).

- Magnetic field: \( B = \nabla \Phi \times \nabla \theta + \nabla \zeta \times \nabla \psi = \nabla \alpha \times \nabla \psi \),

\[ \text{(2)} \]

with \( \alpha = \zeta - q \theta \).

- Parametrize \( z \): Start with
\[ z = (\alpha, (e/c) \psi; s, p_{||} \equiv M v_{||}; \theta_g, J_g \equiv (M/c) \mu) , \]

\[ \text{(3a)} \]

with \( s \equiv \text{distance along } B, (\theta_g, J_g) = \text{gyro-phase & action. Transform} \)
\( (s, p_{||}) \) to \( (\theta_b, J_b) = \text{bounce-phase & action:} \)
\[ z = (\theta, J), \ \theta = (\bar{\alpha}, \theta_b, \theta_g), \ J = (p_\alpha \equiv (e/c) \bar{\psi}, J_b, J_g) \]

\[ \text{(3b)} \]

- For \( H_1 \neq 0 \),
\[ \dot{J}_b = - \partial_{\theta_b} H_1 = - i \sum_{1,m} l_b H_{1,m} \exp i (l \cdot \theta - \omega_m t) \],
\[ \text{(4a)} \]
\[ \dot{E} = \partial_t H_1 = - i \sum_{1,m} \omega_m H_{1,m} \exp i (l \cdot \theta - \omega_m t) \],
\[ \text{(4b)} \]

with Fourier amplitudes \( H_{1,m}(J) \),
\( J \equiv (p_\alpha, J_b, J_g) \), \( \theta \equiv (\bar{\alpha}, \theta_b, \theta_g) \), \( l \equiv (l_\alpha, l_b, l_g) \).

- Diffusion coef \( D(J) \) in \( J \)-space due to \( H_1 \),
\[ D(J) = \sum_{1,m} l l \pi \delta (l \cdot \Omega - \omega_m) \left| H_{1,m}(J) \right|^2. \]
\[ \text{(5)} \]

with \( \Omega(J) \equiv \partial_J H_0 \equiv (\Omega_\alpha, \Omega_b, \Omega_g) \), \( l \equiv (l_\alpha, l_b, l_g) \).
For these \( \omega_m \), have \( l_g=0, \ l_\alpha\to n_\alpha \), and
\( l_b=0, \pm 1, \pm 2, \ldots \) (6a)
- Expect appreciable effect when resonance condition of phase \( l\cdot\theta-\omega_m t \) met:
\[
0=d_t (l\cdot\theta-\omega_m t) = l\cdot\Omega - \omega_m, \quad (6b)
\]
- Projections of \( D(J) \) yield expressions for the various effects noted above, eg,
  - contrib to radial diffusion from \( e^\Psi=\partial_j \Psi \):
    \[
    D^\Psi = e^\Psi \cdot D \cdot e^\Psi = \sum_{l,m} n_\alpha^2 2\pi \delta (l\cdot\Omega - \omega_m) \left| H_{1,m}(J) \right|^2,
    \]
  - energy scattering from \( e^E=\partial_j H_0=\Omega \):
    \[
    D^E = e^E \cdot D \cdot e^E = \sum_{l,m} \omega_m^2 2\pi \delta (l\cdot\Omega - \omega_m) \left| H_{1,m}(J) \right|^2, \quad (7a)
    \]
  - pitch-angle scattering from \( e^J=\partial_j J \):
    \[
    D^J = e^J \cdot D \cdot e^J = \sum_{l,m} l_b^2 2\pi \delta (l\cdot\Omega - \omega_m) \left| H_{1,m}(J) \right|^2 \quad (7c)
    \]
\( \sim \nu_{an} \).
- Preliminary eval’ns of this:

\[ D_{\text{anl}}^{\text{J}} \text{ vs } \alpha_\omega \]

\[ D_{\text{num}}^{\text{J}}(\lambda=0.15) \]

- Assuming \( D \sim 1/\nu_{\text{ef}} \), compare \( D_{\text{num}} \) with analytic expectation:

\[ D_{\text{num}} \text{ vs } \alpha_\omega \]

\[ D_{\text{anl}}/D_0 = \nu/ (\nu + \nu_{\text{an}}) \]

\[ \lambda=0.31 \]
\[ \lambda=0.26 \]
\[ \lambda=0.21 \]
\[ \lambda=0.15 \]

\( \alpha_{\text{E}}=0, \text{S2} \)
Summary:

-A perturbing ES spectrum affects radial transport differently for tokamaks and stellarators. However, for both, the spectrum produces an effective collisionality $\nu_{ef} = \nu + \nu_{an}$, which enters differently into the radial transport.

-Since $D \sim \nu_{ef} = \nu + \nu_{an}$ in tokamaks, the superposition picture $D = D_{nc} + D_{an}$ is also consistent with the $\nu_{ef}$ picture.

-$D_{an}$ in stellarators displays a more complex dependence, exhibiting an oscillatory structure as a function of mode frequency $\omega$ out to larger values of $\omega$.

-For some $\nu$ and $\omega$, the fluctuations can REDUCE $D$ below $D_{nc}$, contrary to the superposition intuition, but consistent with the $\nu_{ef}$ expectation in the $1/\nu$ regime.

-An analytic theory for $\nu_{ef}$ has been developed, providing a prediction for $\nu_{ef}$, and better understanding of the numerical results.