The effect on neoclassical transport of a fluctuating electrostatic (ES) spectrum

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We consider how transport changes in toroidal devices when one superposes on the background magnetic field **B** a specified spectrum $\{\varphi_k\}$ of electrostic (**ES**) modes, representing turbulence, or an externally-applied **E**-field.

-**2 intuitive pictures** for the effect:

(1) Additive (superposition) picture:
Commonly assumed that total diffusion coefficient D is a sum of neoclassical and anomalous contributions,
$D=D_0^{nc}+D^{an}$,
eg, with $D^{an} \sim |\varphi|$ (strong turbulence), $D^{an} \sim |\varphi|^2$ (weak turbulence, quasilinear theory, some ripple transport).

(2) ν_{ef} picture:
One might instead expect the fluctuations to enhance the total effective collisionality $\nu_{ef} = \nu + \nu_{an}$ over the purely collisional rate ν, shortening the decorrelation time.

Configurations:

-We study the transport in 3 configurations:

(1)7q1_tok = tokamak obtained from **7q1** by taking all Fourier components B_{mn} of magnetic field strength =0 for $n \neq 0$.

-Plot $B(\theta,\zeta)$ and B(along field line):

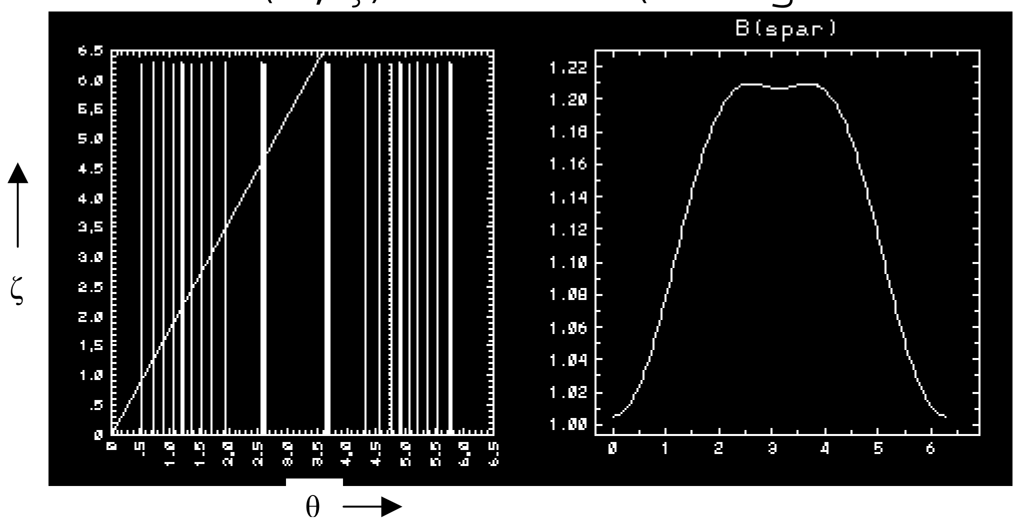


Fig.1

(2)7q1 = one variant of the quasiaxisymmetric stellarator LI383, on which NCSX is based.

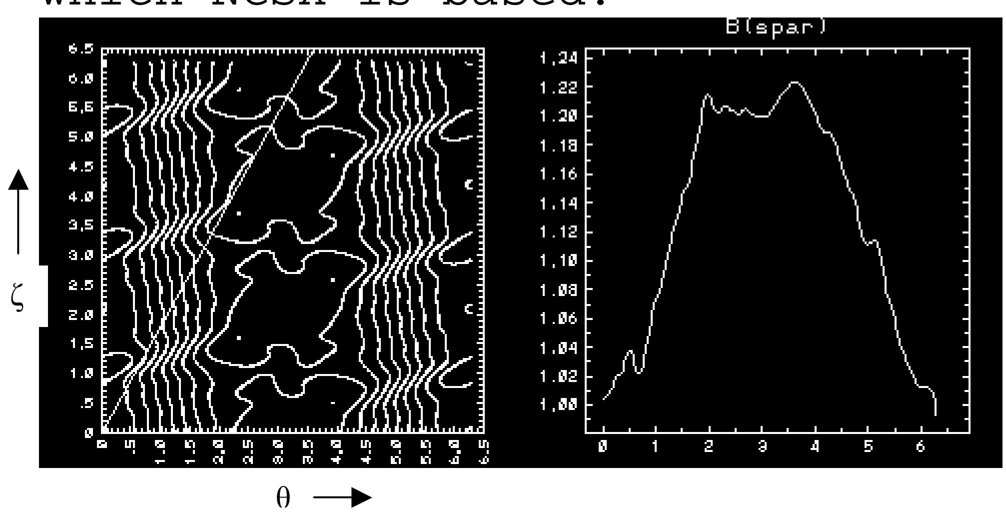


Fig.2

(3)27j = conventional (m,n)=(2,6) stellarator.

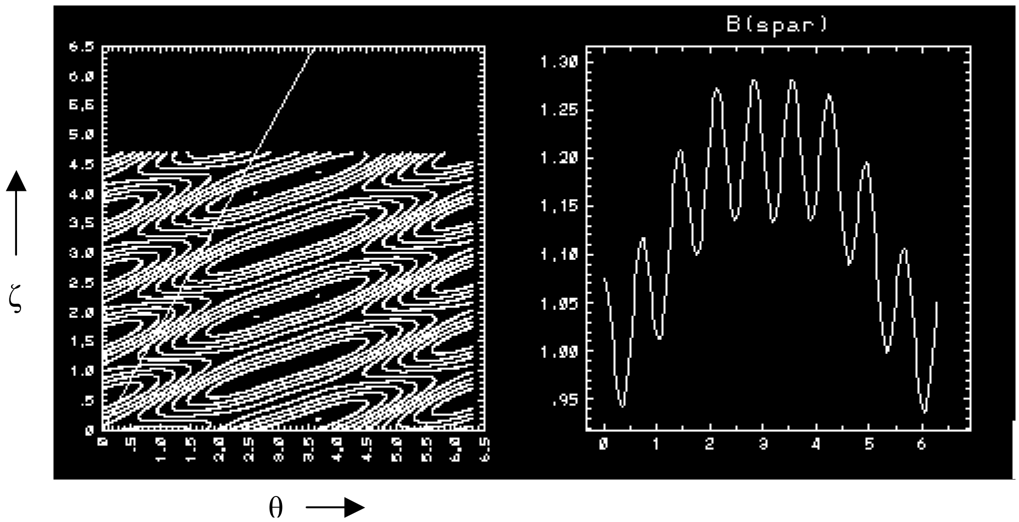


Fig.3

-Ambipolar electric field E_r :

$$E_r = -\partial_r \varphi_0 \quad , \quad \varphi_0 = \alpha_E (1- \psi/\psi_a) \approx \alpha_E (1-r^2/a^2).$$

Perturbing Spectra:

-All configs have q ϵ [2.53, 1.51] ≈[5/2,3/2].

-**Spectrum S1**: Model turbulence with a small spectrum of low-n modes with $q_{mn} \equiv m/n$ in this range: m/n={3/2,5/3,2/1,4/2,6/3,5/2}, with drift-wave (**DW**)-like frequencies,

$\omega_{mn} = \alpha_\omega\ \omega_{*k}/(1+k_\perp^2\rho^2)$,

amplitudes $e\varphi_{mn}/E_1 = \hat{a}_m A_m(\psi)$, with $E_1 \equiv 1$ keV, $\max(A_m(\psi))=1$, $\hat{a}_m \equiv 10^{-3}\ \alpha_A/(1+k_\perp^2\rho^2)$, with α_ω, α_A multiplicative parameters, scanned in numerical studies.

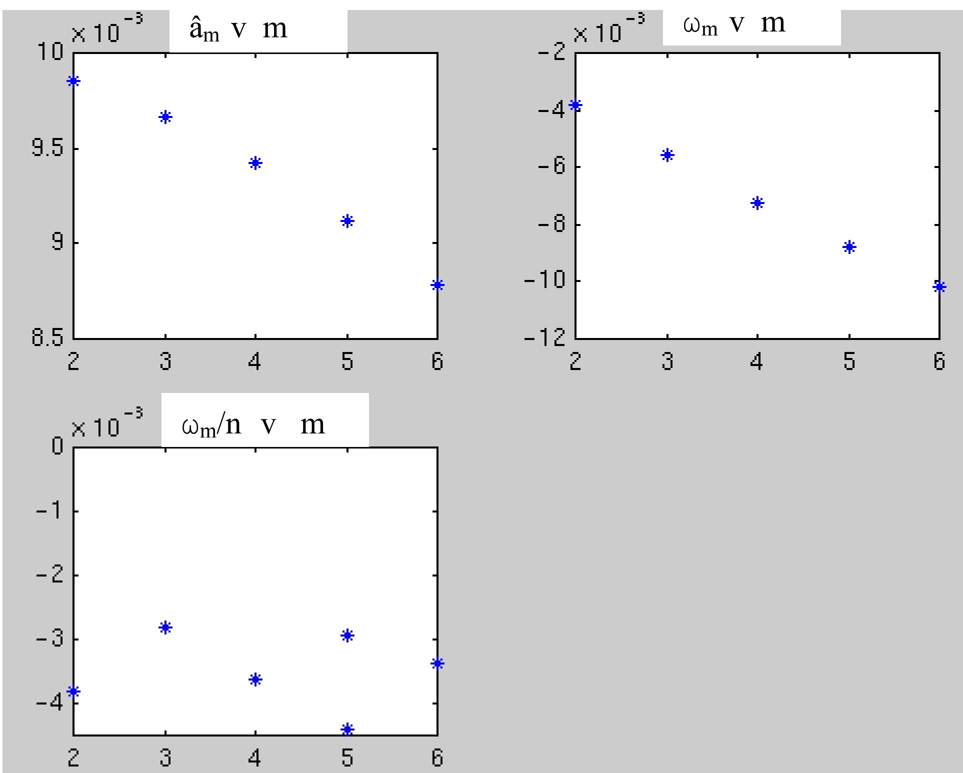


Fig.4

-**Spectrum S2**: As **S1**, but take all n=0.

-Spectrum **S2** has larger $k_{||}$ ⇒ larger $E_{||}$ ⇒ enhanced capacity to break bounce-action J_b, energy E, and so enhance ν_{ef}.

-**S2** models externally-applied RF fields, such as employed on the Saturn stellarator[1] to detrap electrons[1]:

[1] V.S. Voitsenya, et al., *Sov.J.Plasma Phys.* **3**, 659 (1977).

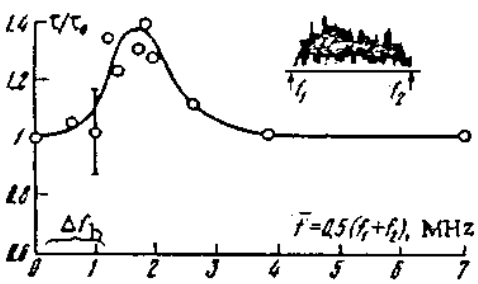


FIG. 4. The ratio τ/τ_0 as a function of the central frequency $\bar{f} = 1/2 \cdot (f_1 + f_2)$; Δf_b is the calculated range of reflection frequencies for the localized electrons found from Fig. 1.

-More recently, some numerical studies have considered possible applications of externally-applied fields, detrapping electrons to control E_r,[2] entrapping ions for impurity removal[3].

[2]Motojima, Shishkin, et al, Nucl.Fusion (2000).
[3]Antufyev, Shishkin, Fusion Science & Tech (2004).

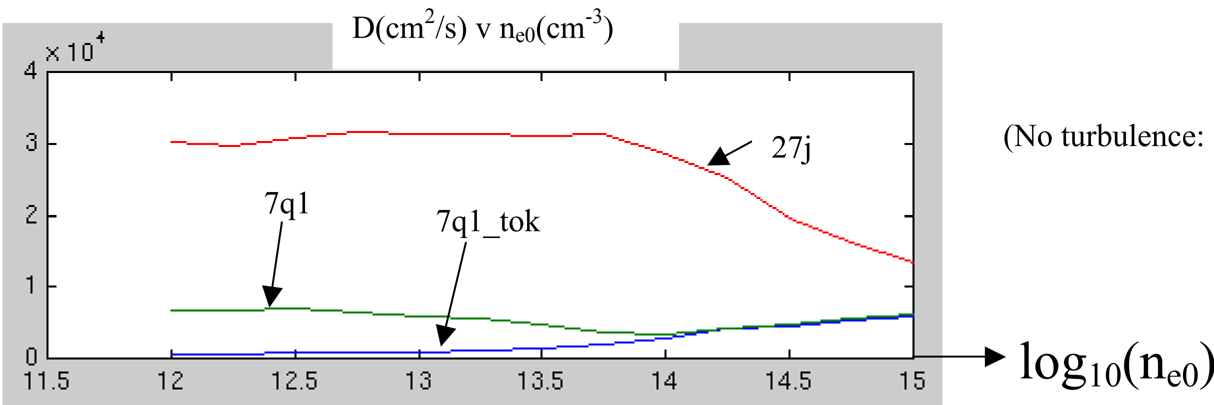
Simulations:

-With background fields $\mathbf{B}(\mathbf{x})$, use GC code ORBIT to integrate the orbits of N_p particles, taking a monoenergetic distribution of hydrogen ions with energy E_0= 1 keV, launched halfway out $[r/a \equiv (\psi/\psi_a)^{1/2}]$in a machine with major radius R_0=1 m, with B_0 (=$|\mathbf{B}|$ on axis of 3 Tesla.

-Compute diffusion coef D from
$D = \langle (\delta r_i)^2 / 2\tau_i \rangle$,
where $\langle F \rangle \equiv N_p^{-1} \Sigma_i F_i$ is an avg over all N_p particles, $\delta r_i \equiv r_i - \langle r \rangle$, and τ_i is the run time for particle i, the smaller of its confinement time and a max run time T.
-Take N_p=3000, unless otherwise noted.

(1) Take radial ambipolar field $E_r = 0$, & spectrum **S1**:

-Scan in collisionality ν:



(No turbulence: α_A=0)

Fig.5

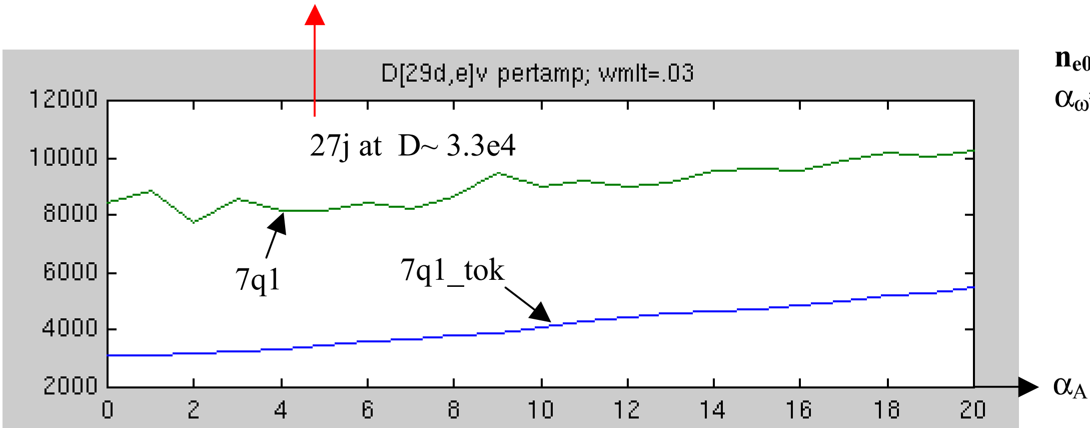
-Banana -> plateau regimes appear in **7q1_tok**.

-**7q1** manifests modest 1/ν regime, coalescing with **7q1_tok** curve at higher n_{e0}.

-**27j** shows appreciable 1/ν regime, as one expects for its much larger ripple.

-Scan in pert amplitude α_A:

-Choose $\mathbf{n_{e0}}$= 10^{13}/cm^3, bit below onset of 1/ν regime in Fig.5.



$\mathbf{n_{e0}}$= 10^{13}/cm^3,
α_ω=wmlt=.03

Fig.6

-Effect of α_A on tokamak consistent with both superposition and ν_{ef} pictures.

-Less effect on stellarator 27j on avg, consistent with ν_{ef} picture. Also, shows more structure than for tokamak.

-Subtracting off α_A=0 contribution (from Fig.5):

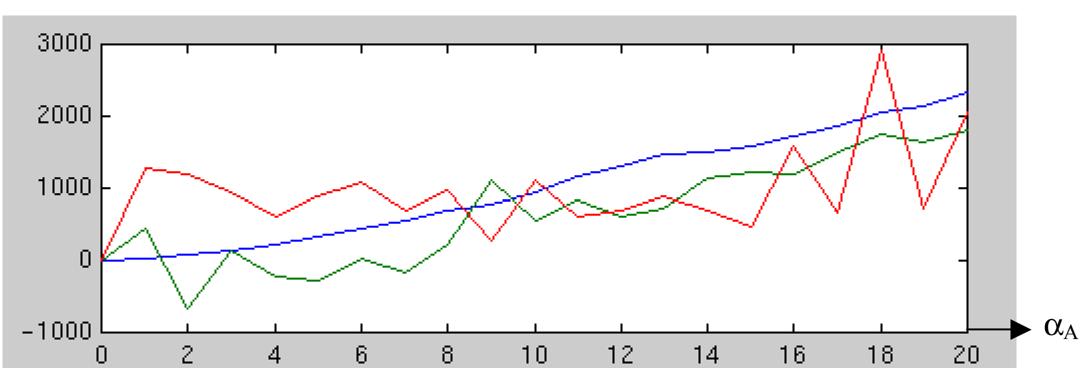
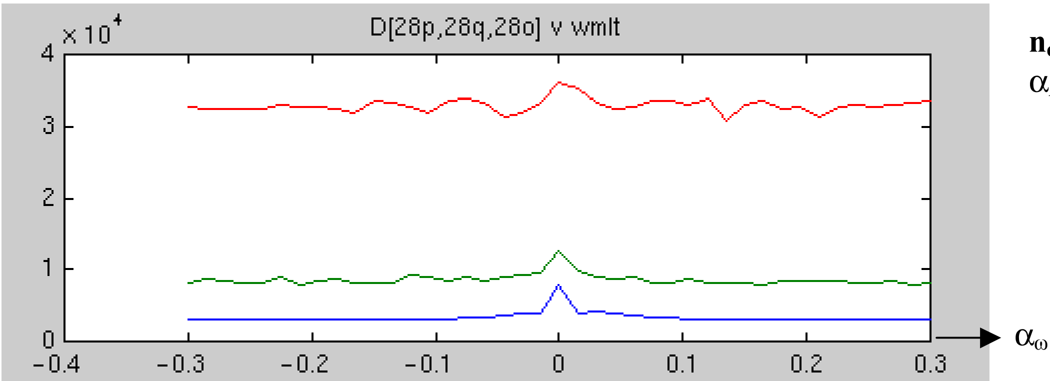


Fig.7

-Scan in frequency (α_ω):



$\mathbf{n_{e0}}$= 10^{13}/cm^3,
α_A = 10

Fig.8

-Again subtracting off α_A=0 contribution:

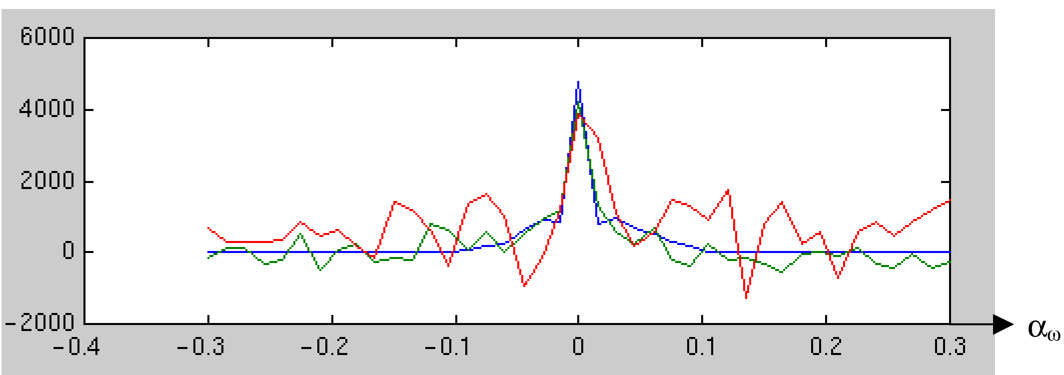


Fig.9

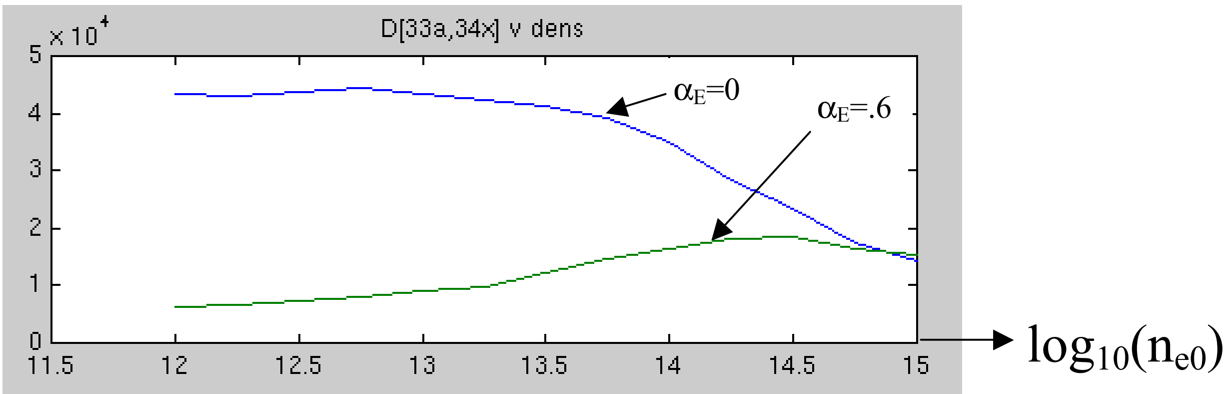
-**7q1_tok** has single central peak of halfwidth $\delta\alpha_\omega \approx .03$.
-**7q1** roughly follows 7q1_tok curve, plus additional structure at larger α_ω.
-**27j** manifests 2 significant features:

(a) The structure seen in 7q1 is more pronounced in 27j, and shows a succession of peaks, with rough spacing $\Delta\alpha_{\omega} \approx .08$.

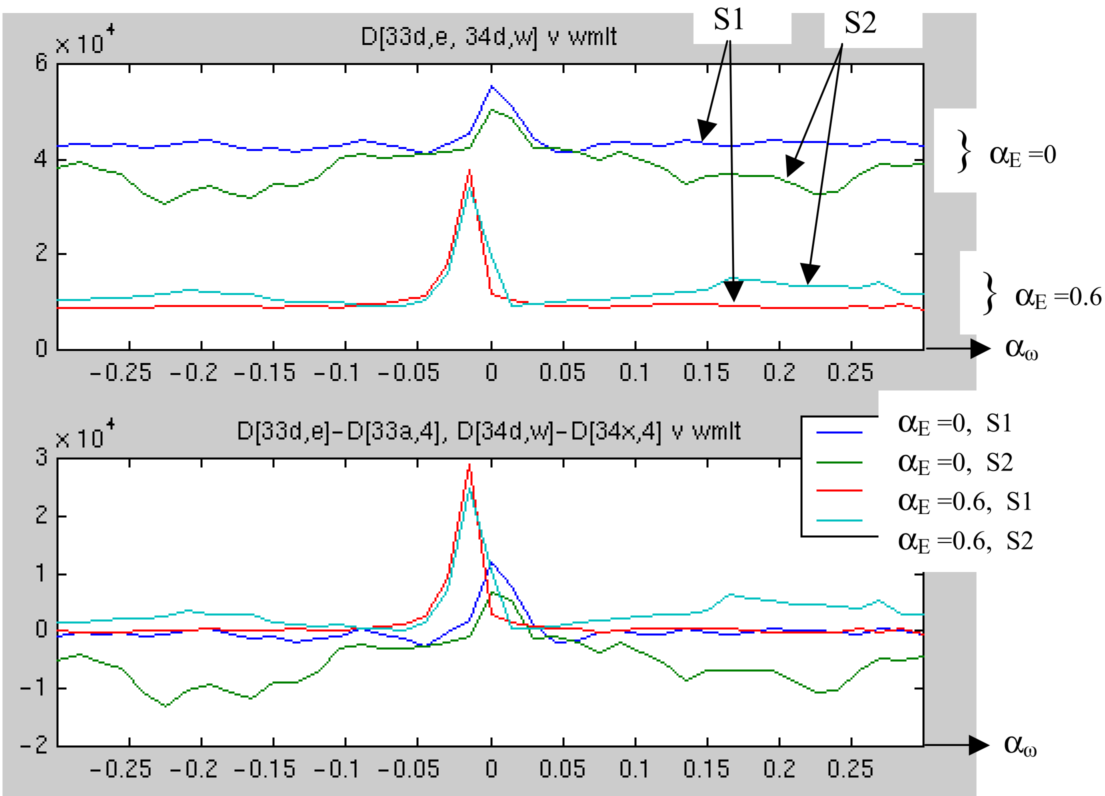
(b) For some α_{ω}, the DW spectrum can REDUCE D[27j] *below* its $\alpha_A=0$ value.

(2) Now, compare $E_r=0$ and $E_r \neq 0$, with spectra **S1,S2**. Focus on **27j** henceforth:

-Scan in ν:



-Frequency scan (α_ω):



-Spectrum **S2** produces larger effect than **S1**, as expected.

-For α_E=0 (puts ions in 1/ν regime), see D^{an} <0 .

-For α_E=0.6 (puts ions in lower-ν "superbanana regime"), see D^{an} >0 .

-Both results what expect for spectrum enhancing ν_{ef}.

Some Theory:

-Kinetic eqn: $(\partial_t + L_H) f = Cf$, (1)
with Hamiltonian $H(\mathbf{z}) = H_0 + H_1$, $L_H \equiv \dot{z}^i \partial_i$,
$\mathbf{z} \equiv \{z^i\}$ (i=1-6) =parametrizing phase-space,
H_0≡unperturbed H, given by background $B(\mathbf{x})$,
and $H_1 = \Sigma_{\mathbf{m}} e\varphi_{\mathbf{m}} \cos\eta_{\mathbf{m}} \sim \alpha_A$, $\mathbf{m} \equiv (m,n)$, $\eta_{\mathbf{m}} \equiv n\zeta - m\theta - \omega_{\mathbf{m}} t$.

-Neoclassical theory follows from (1) with $H_1 \sim \alpha_A \rightarrow 0$.

-Magnetic field: $\mathbf{B} = \nabla\Phi\times\nabla\theta + \nabla\zeta\times\nabla\psi = \nabla\alpha\times\nabla\psi$, (2)
with $\alpha \equiv \zeta - q\theta$.

-Parametrize **z:** Start with
$\mathbf{z} = (\alpha, (e/c)\psi; s, p_{||} \equiv Mv_{||}; \theta_g, J_g \equiv (Mc/e)\mu)$, (3a)
with s≡distance along **B**, (θ_g, J_g) = gyro-phase & action. Transform $(s, p_{||})$ to (θ_b, J_b) = bounce-phase & action:
$\mathbf{z} = (\boldsymbol{\theta}, \mathbf{J})$, $\boldsymbol{\theta} = (\bar{\alpha}, \theta_b, \theta_g)$, $\mathbf{J} = (p_\alpha \equiv (e/c)\bar{\psi}, J_b, J_g)$ (3b)

-For $H_1 \neq 0$,

$$\dot{J}_b = -\partial_{\theta b} H_1 = -i\Sigma_{\mathbf{l},\mathbf{m}} l_b H_{\mathbf{l},\mathbf{m}} \exp i(\mathbf{l}\bullet\boldsymbol{\theta} - \omega_{\mathbf{m}} t), \quad (4a)$$

$$\dot{E} = \partial_t H_1 = -i\Sigma_{\mathbf{l},\mathbf{m}} \omega_{\mathbf{m}} H_{\mathbf{l},\mathbf{m}} \exp i(\mathbf{l}\bullet\boldsymbol{\theta} - \omega_{\mathbf{m}} t), \quad (4b)$$

with Fourier amplitudes $H_{\mathbf{l},\mathbf{m}}(\mathbf{J})$,
$\mathbf{J} \equiv (p_\alpha, J_b, J_g)$, $\boldsymbol{\theta} \equiv (\bar{\alpha}, \theta_b, \theta_g)$, $\mathbf{l} \equiv (l_\alpha, l_b, l_g)$.

-Diffusion coef $\mathbf{D}(\mathbf{J})$ in **J**-space due to H_1,

$$\mathbf{D}(\mathbf{J}) = \Sigma_{\mathbf{l},\mathbf{m}} \mathbf{l}\mathbf{l}\pi\delta(\mathbf{l}\bullet\boldsymbol{\Omega} - \omega_{\mathbf{m}}) |H_{\mathbf{l},\mathbf{m}}(\mathbf{J})|^2. \quad (5)$$

with $\boldsymbol{\Omega}(\mathbf{J}) \equiv \partial_{\mathbf{J}} H_0 \equiv (\Omega_\alpha, \Omega_b, \Omega_g)$, $\mathbf{l} \equiv (l_\alpha, l_b, l_g)$.

For these $\omega_{\mathbf{m}}$, have $l_g=0$, $l_\alpha \rightarrow n_\alpha$, and

$l_b=0,\pm 1,\pm 2,..$ (6a)

-Expect appreciable effect when resonance condition of phase $\mathbf{l}\bullet\theta-\omega_{\mathbf{m}}t$ met:

$$0=d_t(\mathbf{l}\bullet\theta-\omega_{\mathbf{m}}t)= \mathbf{l}\bullet\mathbf{\Omega}-\omega_{\mathbf{m}} , \qquad (6b)$$

-Projections of $\mathbf{D}(\mathbf{J})$ yield expressions for the various effects noted above, eg,

-contrib to radial diffusion from $\mathbf{e}^{\Psi}\equiv\partial_{\mathbf{J}}\Psi$:

$$D^{\Psi\Psi}=\mathbf{e}^{\Psi}.\mathbf{D}.\mathbf{e}^{\Psi}=\Sigma_{\mathbf{l},\mathbf{m}}n_\alpha^2\pi\delta(\mathbf{l}\bullet\mathbf{\Omega}-\omega_{\mathbf{m}})|H_{\mathbf{l},\mathbf{m}}(\mathbf{J})|^2 ,$$

-energy scattering from $\mathbf{e}^{E}\equiv\partial_{\mathbf{J}}H_0=\mathbf{\Omega}$: (7a)

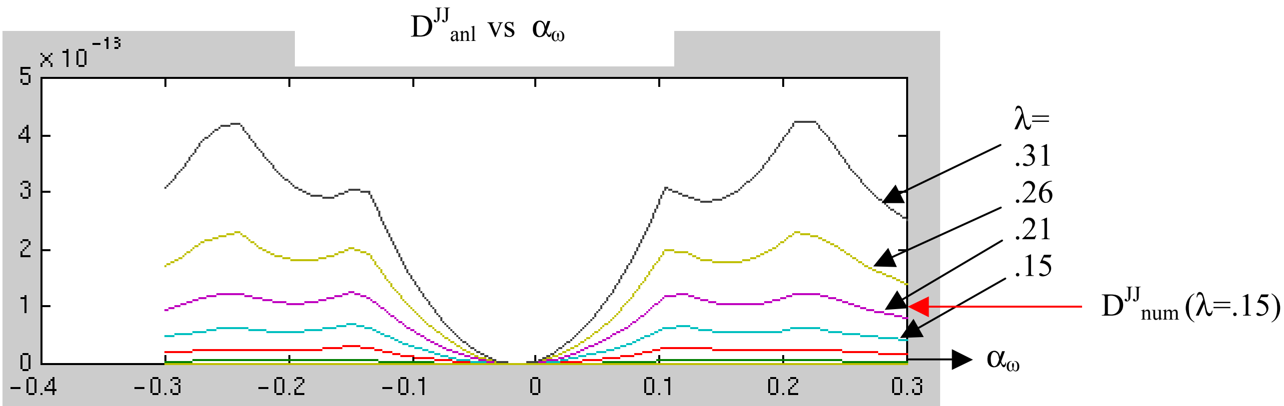
$$D^{EE}=\mathbf{e}^{E}.\mathbf{D}.\mathbf{e}^{E}=\Sigma_{\mathbf{l},\mathbf{m}}\omega_{\mathbf{m}}^2\pi\delta(\mathbf{l}\bullet\mathbf{\Omega}-\omega_{\mathbf{m}})|H_{\mathbf{l},\mathbf{m}}(\mathbf{J})|^2 , \qquad (7b)$$

-pitch-angle scattering from $\mathbf{e}^{J}\equiv\partial_{\mathbf{J}}J$:

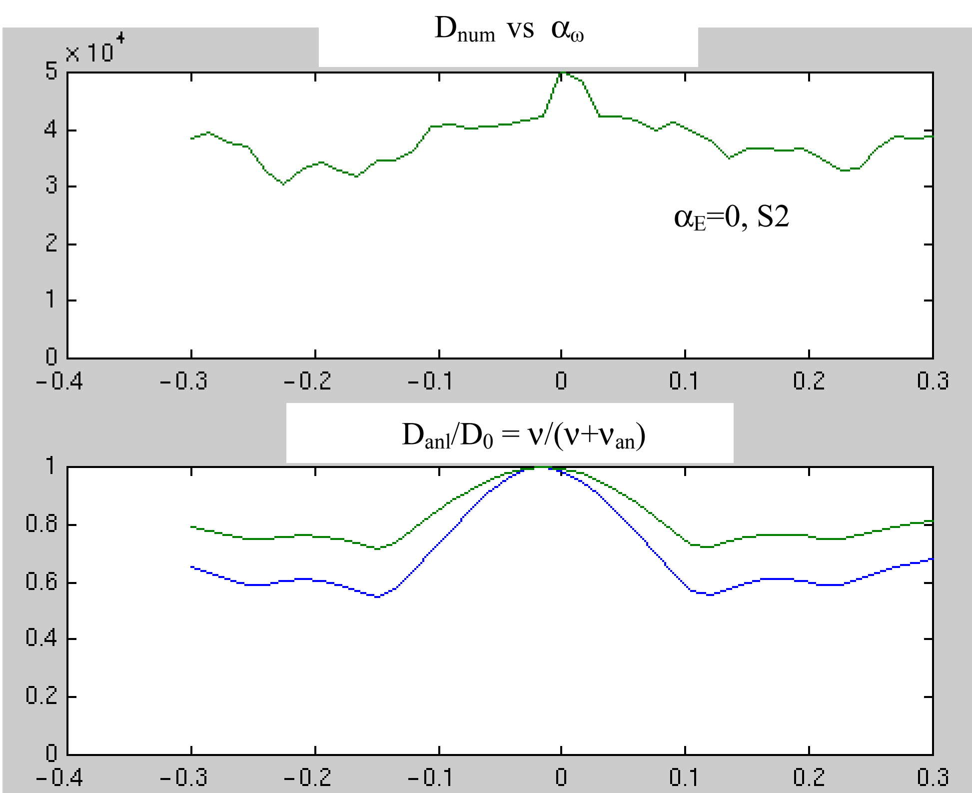
$$D^{JJ}=\mathbf{e}^{J}.\mathbf{D}.\mathbf{e}^{J}=\Sigma_{\mathbf{l},\mathbf{m}}l_b^2\pi\delta(\mathbf{l}\bullet\mathbf{\Omega}-\omega_{\mathbf{m}})|H_{\mathbf{l},\mathbf{m}}(\mathbf{J})|^2 \qquad (7c)$$

$\sim \nu_{an}$.

-Preliminary eval'ns of this:



-Assuming D~ $1/\nu_{ef}$, compare D_{num} with analytic expectation:



Summary:

-A perturbing ES spectrum affects radial transport differently for tokamaks and stellarators. However, for both, the spectrum produces an effective collisionality $\nu_{ef} = \nu + \nu_{an}$, which enters differently into the radial transport.

-Since $D \sim \nu_{ef} = \nu + \nu_{an}$ in tokamaks, the superposition picture $D = D^{nc} + D^{an}$ is also consistent with the ν_{ef} picture.

-D^{an} in stellarators displays a more complex dependence, exhibiting an oscillatory structure as a function of mode frequency ω out to larger values of ω.

-For some ν and ω, the fluctuations can REDUCE D below D^{nc}, contrary to the superposition intuition, but consistent with the ν_{ef} expectation in the $1/\nu$ regime.

-An analytic theory for ν_{ef} has been developed, providing a prediction for ν_{ef}, and better understanding of the numerical results.