Investigation of the High, Finite n Ballooning Mode Limit for Compact Quasi-Axially Symmetric Stellarators

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OUTLINE

• Ballooning stability of QAS
• Global MHD code comparisons: CAS3D, TERPSICHORE
• Kink and vertical stability robustness
• QAS stability without a nearby conducting wall
• Global MHD calculations near the ballooning limit
• Conclusions
MHD STABILITY OF NCSX

- Candidate designs for the National Compact Stellarator Experiment (NCSX) being examined
- **QAS3_C82 (EPS’99):**
  - Good physics properties
  - but high coil currents => difficult engineering
- **QAS3_LI383 (IAEA’00)**
  - Better physics properties and several good, improved machine coil designs
Iota is higher with less shear and the pressure profile is broader in LI383 than in the earlier design C82.

Higher iota also denotes lower q so that particle loss rates will be lower.

Present stellarator experiments (CHS, LHD) have broader pressure profiles than C82.
Iota Profiles of Previous and Present Designs for National Compact Stellarator Experiment

High fraction of field transform from stellarator coils
Published Pressure Profiles for Advanced Tokamak and Stellarator Experiments

Peaked central pressures found for DIII-D and W7-AS with ECH heating.

Broad pressures characteristic of CHS and LHD and as for LI383 design.

Compiled by David Mikkelson, PPPL
Boundary Shapes of Previous and Present Designs for National Compact Stellarator Experiment

Three field period quasiaxially symmetric stellarator, views spaced 20 degrees apart

Drift-orbit optimized stellarator design: tokamak particle orbits
Ballooning Stability of QAS
At 4% Beta TERPSICHORE Module VVBAL Predicts Mercier Instability and Marginal Ballooning Stability for QAS3_LI383

Balloonning Eigenvalues

Mercier Criterion

iota=0.6

iota=0.6667
COBRA Code Ballooning Stability Calculations for LI383 Show Stability at Beta = 4.0%, Instability at Beta=4.25%

TERPSICHERE Predicts Marginal Stability for QAS3_LI383 at 4% Beta
Ballooning Mode Eigenvalues in QAS3_LI383
Show Strong Dependence on Field Line
due to Fully Three-dimensional Symmetry of the Torus

Tokamak Exhibits Monotonic Behavior of the Eigenvalue
with Choice of Field Line, $\alpha = \zeta + q^*\theta$, at 4.3% Beta

$\Rightarrow$ toroidal localization of ballooning instability: Anderson localization

Maximum eigenvalue versus alpha for flux surfaces between 0.92 and 0.95.

TERPSICHERE ballooning calculations
were made at alpha = pi/2, where eigenvalues are minimal.
TERPSICHEREE ballooning instability for new NCSX design, in range $\Delta s$ ($s$ = toroidal flux).

VMEC equilibria with pressure and current scaled to increase beta. Shown here are the results calculated at 129 flux surfaces.

### QAS3-LI383

<table>
<thead>
<tr>
<th>Beta (%)</th>
<th>$\Delta s$</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.251</td>
<td>.87-.94</td>
<td>■</td>
</tr>
<tr>
<td>5.110</td>
<td>.76-.94</td>
<td>■</td>
</tr>
<tr>
<td>5.971</td>
<td>.73-.94</td>
<td>■</td>
</tr>
<tr>
<td>6.833</td>
<td>.72-.94</td>
<td>■</td>
</tr>
<tr>
<td>7.695</td>
<td>.68-.94</td>
<td>■</td>
</tr>
<tr>
<td>8.557</td>
<td>.73-.94</td>
<td>■</td>
</tr>
</tbody>
</table>

Ballooning Eigenvalues ($-\omega^2$)
TERPSICORE ballooning instability for previous NCSX design, in range $\Delta s$ ($s =$ toroidal flux).

VMEC equilibria with pressure and current scaled to increase beta. Shown here are the results calculated at 129 flux surfaces.

**QAS3-C82**

<table>
<thead>
<tr>
<th>Beta (%)</th>
<th>$\Delta s$</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.862</td>
<td>.79-.97</td>
<td>■</td>
</tr>
<tr>
<td>4.630</td>
<td>.80-.98</td>
<td>■</td>
</tr>
<tr>
<td>5.393</td>
<td>.72-.98</td>
<td>■</td>
</tr>
<tr>
<td>6.153</td>
<td>.71-.99</td>
<td>■</td>
</tr>
<tr>
<td>6.908</td>
<td>.63-.99</td>
<td>■</td>
</tr>
<tr>
<td>7.659</td>
<td>.54-.99</td>
<td>■</td>
</tr>
</tbody>
</table>
Global MHD code
Comparisons: CAS3D, TERPSICHORE
TERPSICHERE and CAS3D Solve Variational Equation

$$\delta W_p + \delta W_v - \omega^2 \delta W_k = 0.$$ 

potential energy in the plasma,

magnetic energy in the vacuum region,

kinetic energy and the eigenvalue of the system.

The MHD perturbations evolve as exp (i\(\omega t\)), being unstable if \(\omega < 0\).

Calculate the normal displacements of the unstable eigenfunction \(\xi\) and the plasma potential energy change \(\delta W\).
CAS3D Code Package Calculations
Based on the plasma potential energy

\[ W_p = \frac{1}{2} \iiint d^3r \left[ |C|^2 - A(\xi \cdot \nabla s)^2 + \gamma p(\nabla \cdot \xi)^2 \right] \]

associated with the displacement \( \xi \).

Vector \( C \), stabilizes plasma energy integral.
\( C^1 \) describes field line bending energy
\( C^2 \) depends on local shear and parallel current density
\( C^3 \) is field compression energy.

**Destabilization** is driven by the second term in \( W_p \),
with the current density \( j \) in \( A \)

\[ A = 2|\nabla s|^4 (j \times \nabla s) \cdot (B \cdot \nabla) \nabla s \]

driving instability, modulated by the plasma curvature and the local shear.

The **third term in \( W_p \) is stabilizing**, proportional to \( \gamma \);
\( \gamma \) is the ratio of the specific heats and describes the energy associated with field compression.
The code version used here is for incompressible modes
\( (\nabla \cdot \xi = 0) \)
The stabilizing term proportional to \( \gamma p \) does not contribute.
Comparison of TERPSICHERE and CAS3D
Calculations of Kink Instability for Peaked Pressure in C82

Shapes and identification of largest harmonic components in excellent agreement.
QAS Stability without a Conducting Wall
The value of the marginal beta below which the kink is stable depends on the distance between the plasma and the conducting wall for QAS as for tokamaks.
NCSX: Toroidal Plasma with Helical Coils for Passive Control
Pressure and Iota Profiles
Varied about the Design Point Configuration
To test robustness of stability and confinement of QAS configuration
Unstable global mode
Unstable edge mode
Stable

P02 drives global kink mode for \( \iota(a) > 0.5 \).
Global mode thought to be stabilized by position of the rational q surface with respect to the configuration profiles.

N=1, External kink stability in general, increases with lower pressure gradient and higher magnetic shearat edge.

Non-Periodicity-Preserving N=1 Mode
Kink Stability Results Summary for NCSX Maintaining \( \beta \geq 3.8\% \) and Boundary Shape
N=0 Periodicity-preserving mode stability increases with lower pressure gradient and higher magnetic shear; More regions of "vertical" than kink stability.

\[ \beta \geq 3.8\% \]

\[ \nabla \iota \bigg|_a \]

\[ \nabla P \bigg|_a \]

QAS3_C82 Design point

Unstable global mode

Unstable edge mode

Stable

\[ \iota(a) > 0.5 \]
Fast Particle Loss Fraction for QAS3_C82, D Beams, 1T Shows Little Dependence on Pressure and Iota Configuration

At 2T, hydrogen beam loss with full collision model ~25% energy loss.
Global MHD Calculations near the Ballooning Limit
GLOBAL, 3D MHD: REDUCED BETA LIMIT WITH INCREASED MAXIMUM TOROIDAL MODE OF UNSTABLE EIGENFUNCTION

CALCULATIONS WITH TERPSICHORE FOR QAS3_C82
Ballooning Beta Limit

- Beta limit for ballooning mode stability: radially local, infinite $n$
- Density of states argument based on quantum chaos (R. Dewar see WP1.088)
  Basis functions with maximum $n_{\text{toroidal}} \sim 60$
  needed for ballooning limit (for H1 heliac).
- Kinetic effects on the ballooning limit: finite Larmor radius and diamagnetic ion frequency.
- Finite Larmor radius:
  For the NCSX, $B \sim 1T$, hydrogen plasmas, $T_e(0) \sim 2kev$, $T_e(a) \sim 0.5kev$
  - Larmor radius is 0.7 cm at the center and 0.3cm near the plasma edge.
  - The CAS3D high $n$ Mercier modes at $n_{\text{toroidal}} = 70$ are $\sim 2$cm.
  Unless modes at high $n$ have much reduced widths, stabilization by kinetic effects on radial width does not seem likely.
- Diamagnetic ion frequency effects on ballooning limit difficult to estimate, because of CAS3D energy normalization
- Rewoldt has shown that kinetic stabilization can change the mode growth rates, but not the beta limit, for tokamaks.
CAS3D Calculations of Mercier, High n MHD

- High n: designate \((M_{\text{BIG}}, N_{\text{BIG}})\) centroid of basis functions: \((11,-7), (44,-28), (110,-70)\)
- Initial results for natural resonances: Mercier modes near \(\iota = 0.6\)
- Radial extent of eigenfunctions decreases as \(N_{\text{BIG}}\) increases: radial localization
- Work in progress without natural resonances, for QAS3_LI383
CAS3D CALCULATIONS
FOR QAS3_LI383

AT 4.3% BETA: INCREASING TOROIDAL MODE NUMBER REDUCES RADIAL EXTENT OF MOST UNSTABLE MODE

PRESENT CALCULATIONS, FOR 192 FLUX SURFACES, 70 BASIS FUNCTIONS ABOUT (M_BIG,N_BIG) INCLUDE NATURAL RESONANCES

WORK IN PROGRESS TO EXAMINE BETA LIMIT WITHOUT NATURAL RESONANCES (MERCIER MODES)

- (MBIG,NBIG)=(11,-7)
- (MBIG,NBIG)=(44,-28)
- (MBIG,NBIG)=(110,-70)
Conclusions

- Compact quasiaxial stellarators have been designed to have good particle confinement and good MHD stability
- Ballooning stability is found at 4% beta
- Global MHD calculations are in progress for high n modes, possibly to increase plasma beta
- CAS3D and TERPSICHORE codes are verified to be in good agreement
- Compact QAS designs are kink stable without a nearby conducting wall.
References

- G. Fu, Phys. Plas. 7 (2000) 1079;