Confinement Optimization and Analysis of Compact Stellarators using the DKES Model
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Introduction/Motivation
Optimization Method and Transport Analysis Tools
Physics evaluations for recent configurations
Electron and ion neoclassical losses
Energetic orbit losses
Bootstrap current
Conclusions
Compact stellarators (low $A = R/\langle a \rangle$) offer:

- Combination of the desirable features of **TOKAMAKS** (low $A$, high $\beta$, good confinement)
- with those of the **STELLARATOR** (low recirculating power, disruption avoidance)
- Focus is on **COMPACTNESS** (while preserving confinement/stability)
Features of low A = R/<a> stellarators

- Lower cost near-term experiments while maintaining a similar plasma radius as large aspect ratio devices
- Longer term potential of a more economically-sized, higher-power-density reactor
- Opens up a new regime of stellarator parameter space with new physics expected in:
  - transport
  - equilibrium fragility
  - plasma flow dynamics
  - enhanced confinement regimes
  - RF heating strategies
  - microturbulence
Strategies for QOS Optimization

• Optimize an ultra low aspect ratio \((A = 2.5 - 3)\), low \(\beta\) configuration for a Concept Exploration experiment
  – most of the rotational transform supplied externally

• Optimize compact \((A = 3 - 3.5)\), high \(\beta\) configurations as part of the longer-term QOS program
  – a larger fraction of the transform provided by plasma currents
Methods

• Stellarator optimization
• Transport evaluation
• Energetic Particle Confinement
• Bootstrap Currents
Optimization Process Successfully Integrates a complex, interacting set of Physics Criteria:

<table>
<thead>
<tr>
<th>Targets (Physics/Engineering)</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounce-average omnigeneity (drift surfaces and flux surfaces aligned)</td>
<td>$B_{\text{min}} = B_{\text{min}}(\psi)$, $B_{\text{max}} = B_{\text{max}}(\psi)$, $J = J(\psi)$</td>
</tr>
<tr>
<td>Target nearby quasi-symmetries</td>
<td>Minimize $B_{mn}$ if $m \neq 0$ (QP), or if $m/n \neq 1$ (QH)</td>
</tr>
<tr>
<td>Local diffusive transport</td>
<td>$D$, $\chi$ from DKES</td>
</tr>
<tr>
<td>Current profile</td>
<td>self-consistent $I_{bs}$, $l(\psi)$ goes to 0 at edge</td>
</tr>
<tr>
<td>Limit maximum plasma current</td>
<td>e.g., $I_{\text{max}} &lt; 40$ kAmps</td>
</tr>
<tr>
<td>Iota profile</td>
<td>$i(\psi) = 0.5$ ($\rho = 0$) to 0.8 ($\rho = a$)</td>
</tr>
<tr>
<td>Magnetic Well, Mercier</td>
<td>$V'' &lt; 0$, $D_m &gt; 0$ over cross section</td>
</tr>
<tr>
<td>Ballooning stability</td>
<td>$&lt;\beta&gt; \sim 2$-4%</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>$R_0/a \approx 2.5$ to 3.5</td>
</tr>
<tr>
<td>Limit outer surface curvature</td>
<td>avoid strong elongation/cusps</td>
</tr>
</tbody>
</table>

Control variables:

- shape (30-40 Fourier harmonics $R_{mn}$, $Z_{mn}$) for LCFS + profile parameters
Transport optimizations using the DKES transport target have resulted in confinement improvement.

3 field periods ($A = 3.4$)

$e\phi/kT = 0$

Correlation of DKES and DELTA5D Confinement Times

- DKES evaluated at $s = 0.5$
- DKES evaluated at $s = 0.3$
- DKES evaluated at $s = 0.7$

Monte Carlo global $\tau_{E,ion}$ (msec)
Transport tools

- General purpose stellarator particle simulation code (DELTA5D)
  - thermal electron/ion transport, bootstrap current
  - alpha particles
  - neutral beams, ICRH tails
  - uses MPI to achieve near linear speedup with number of processors
- Drift Kinetic Equation Solver (DKES)
  - variation of bootstrap current with collisionality and electric field
  - local particle fluxes ambipolarity condition
  - integrate over profiles to obtain global lifetimes
  - uses shared memory OpenMP parallelism to achieve ~ x 3 speedup
    (with Ed D’Azevedo, ORNL CCS Division)
- Other qualitative measures: J, B_{min}, B_{max}, |B| contours
The DKES (Drift Kinetic Equation Solver) provides the full neoclassical transport coefficient matrix (multi-helicity)

\[
I_i = \left[ \begin{array}{c} 
\bar{\Gamma} \cdot \bar{\nabla}_s \\
\frac{1}{T} \bar{\Omega} \cdot \bar{\nabla}_s \\
n \langle (\bar{u} - \bar{u}_s) \cdot \bar{B} \rangle 
\end{array} \right] = -\sum_{j=1}^{3} L_{ij} A_j
\]

\[
A_j = \left[ \begin{array}{c} 
\frac{n'}{n} - 3 \frac{T'}{T} - eE_x \\
\frac{T'}{T} \\
\left( \frac{e}{T} \langle \bar{E} \cdot \bar{B} \rangle \right) \langle B^2 \rangle 
\end{array} \right]
\]

\[
L_{ij} = n \frac{2}{\sqrt{\pi}} \int_0^\infty dK \sqrt{K} e^{-K} g_i g_j D_{ij}
\]

where \( g_1 = g_3 = 1, \ g_2 = K, \ K = \left( \frac{V}{V_{th}} \right)^2 \)

\[
D_{11} = D_{12} = D_{21} = D_{22} = -\frac{V_{th}}{2} \left[ \frac{B V_{th}}{\Omega} \left( \frac{d\rho}{dr} \right)^{-1} \right]^2 K \sqrt{K} \Gamma_{11}
\]

\[
D_{31} = D_{32} = D_{13} = D_{23} = -\frac{V_{th}}{2} \left[ \frac{B V_{th}}{\Omega} \left( \frac{d\rho}{dr} \right)^{-1} \right] K \Gamma_{31}
\]

\[
D_{33} = -\frac{V_{th}}{2} \sqrt{K} \Gamma_{33}
\]

\[
\Gamma_{ij} = \Gamma^1_{ij} \left( \frac{V}{\sqrt{\nu}}, \frac{E_x}{\nu} \right)
\]


- Variational: provides upper and lower bounds on \( dS/dt \)

- Expands \( f \) in Fourier-Legendre series

(i.e., to carry out the above integrals, one will need to generate a 2-D matrix of \( \Gamma \)'s vs. these parameters for each flux surface)
DELTA5D Monte Carlo code is used for both thermal plasma and fast ion confinement studies.

- **Thermal plasma**
  - Global and local diffusive limits

- **Various fast ion populations**
  - ICRF tails (quasilinear diffusion operator)
  - Neutral beam ions (pencil beam approximation)
  - Alphas
  - Alfvén turbulence (to be added)

- **Options for f and δf particle weightings**

- **Diagnostics:** particle and energy losses, loss patterns, energy slowing down, escaping pitch angle/energy/lifetime distributions

- **Longer term goal:** Multi-species (thermal, fast ion, impurity), coupled transport and electric field evolution model

- **Computational characteristics**
  - parallelization over groups of particles
  - uses collective MPI communications, runs on T3E and IBM-SP
Configuration Development

- Ultra low A (2.5 - 3), near-term devices
  - most of the transform supplied externally
  - quasi-poloidal symmetry built-in

- Compact (A = 3 - 3.5) high β devices
  - large fraction of transform comes from current
  - quasi-poloidal symmetry enhanced by high β

- CE selection guidelines
  - Compact: A < 3
    This ultra low aspect ratio range is lower than existing stellarators (1/2 to 2/3 that of NCSX)
  - Good confinement: $\tau_{\text{neo}}$ at least $> 2^*\tau_{\text{ISS95}}$
    Drift-optimized neoclassical transport not the dominant loss
  - Stability: MHD stable at $\beta \sim 2\%$
    Ballooning and Mercier analysis included in optimization to ensure stability at $\beta$ levels relevant to a CE

- Accessibility, Flexibility
Quasi-Poloidally Symmetric Cases

- Low aspect ratio: $A < 2.5$
  - Have obtained configurations with aspect ratios in the range: $A=2.1$ to $A=3.0$
- Rotational transform below 0.5: $\iota \sim 0.3 - 0.4$
  - Majority of the transform is from the coils, bootstrap current causes $\iota$ to increase
  - Max. Toroidal Current = 25 - 35 kA for $<\beta>$ in the 1 to 1.5% range
  - Stable to neoclassical tearing modes
- Weak shear with iota mainly from coils

- Bootstrap consistency is fair
Free boundary A2.5_M2_B1.3 configuration (from coils) preserves similar transport as original fixed boundary case:

- green = free boundary
- blue = fixed boundary
Projected QO/CE heating scenarios include both ECH and ICH regimes

<table>
<thead>
<tr>
<th>RF</th>
<th>( P_{\text{heat}} ) (MW)</th>
<th>( &lt;B&gt; ) (Tesla)</th>
<th>( n/10^{20} ) (m(^{-3}))</th>
<th>( T_e ) (keV)</th>
<th>( T_i ) (keV)</th>
<th>( \tau_{\text{ISS95}} ) (msec)</th>
<th>( \nu_{\text{elec}} )</th>
<th>( \nu_{\text{ion}} )</th>
<th>( &lt;\beta&gt; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ECH</td>
<td>0.5</td>
<td>1</td>
<td>0.18</td>
<td>1.4</td>
<td>0.15</td>
<td>8.1</td>
<td>0.019</td>
<td>1.6</td>
<td>0.7</td>
</tr>
<tr>
<td>(2) ECH</td>
<td>1</td>
<td>0.5</td>
<td>0.045</td>
<td>2.1</td>
<td>0.2</td>
<td>1.5</td>
<td>0.0021</td>
<td>0.22</td>
<td>1</td>
</tr>
<tr>
<td>(3) ICH</td>
<td>1</td>
<td>1</td>
<td>0.83</td>
<td>0.5</td>
<td>0.5</td>
<td>11.7</td>
<td>0.68</td>
<td>0.64</td>
<td>2</td>
</tr>
<tr>
<td>(4) ICH</td>
<td>1</td>
<td>0.5</td>
<td>0.59</td>
<td>0.4</td>
<td>0.25</td>
<td>5.5</td>
<td>0.75</td>
<td>1.8</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Model used:

- \( n = \text{constant}, \ Z_{\text{eff}} = 1 \)
- \( (1 - r^2)^2 \ T_e, \ T_i \) profiles
- \( e\phi(r) \) varies inversely with \( kT_e \)
- ion root and electron root investigated
Confinement in the 2 field period, A = 2.5 configuration covers a range from $\tau_{E,\text{global}} = (1.4 \text{ to } 3.6) \tau_{E,\text{ISS95}}$ for different ECRF and ICRF heating scenarios.

<table>
<thead>
<tr>
<th>RF</th>
<th>B (Tesla)</th>
<th>$\tau_{E,\text{ion}}$ (msec)</th>
<th>$\tau_{E,\text{elec}}$ (msec)</th>
<th>$\tau_{E,\text{global}}$ (msec)</th>
<th>$\tau_{E,\text{ISS95}}$ (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECH</td>
<td>1</td>
<td>16.2</td>
<td>17.4</td>
<td>16.2</td>
<td>8.1</td>
</tr>
<tr>
<td>ECH</td>
<td>0.5</td>
<td>4.27</td>
<td>1.95</td>
<td>2.1</td>
<td>1.5</td>
</tr>
<tr>
<td>ICH</td>
<td>1</td>
<td>27</td>
<td>$\sim$100</td>
<td>41.7</td>
<td>11.7</td>
</tr>
<tr>
<td>ICH</td>
<td>0.5</td>
<td>7.7</td>
<td>$\sim$55</td>
<td>16.4</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Monte Carlo energy lifetimes
Energetic particle loss simulations show exit pitch angle, energy and exit position of ions on outer flux surface

DELTA5D Transport analysis
Collisionality and electric field dependence of bootstrap current coefficient (results shown are for $N_{fp} = 2$, $A = 2.5$ device)
Self-Consistent ambipolar electric field calculations
- initially DKES will be used offline for electrons and ion to obtain $\phi(r)$ for DELTA5D
- next step is to use DKES for electron flux coupled with DELTA5D for ion flux

Ion, electron fluxes from DKES

\[
\Gamma_i = -L_{i1}^i(E_i) \left[ \frac{n'}{n} - \frac{3}{2} \frac{T'}{T} - \frac{ZeE_i}{T} \right] - L_{i2}^i(E_i) \frac{T'}{T}
\]

\[
\Gamma_e = -L_{e1}^e(E_e) \left[ \frac{n'}{n} - \frac{3}{2} \frac{T'}{T} + \frac{eE_e}{T} \right] - L_{e2}^e(E_e) \frac{T'}{T}
\]

DKES/DELTA5D Transport analysis

Ion flux from DELTA5D

- Ion flux (T$_i$ = 300eV)
- Ion flux (T$_i$ = 200eV)
- Ion flux (T$_i$ = 100eV)
- Electron flux (T$_e$ = 1000eV)
High $\beta$ Configurations

- A class of configurations with high $\beta$ MHD stability limits
  - Rotational transform primarily from plasma current
  - Proper level of self-consistent bootstrap current and better profile alignment than in advanced tokamaks
  - Stable at higher $\beta$ than comparable tokamak due to lower current
  - Have obtained 3 field period configurations with ballooning stability up to $\beta=23\%$, Vertical/Kink stability up to $\beta=15\%$ (G. Fu), aspect ratios $A\sim 3.5$ - 4.5
High $\beta$ Case: 3 Field Periods

- 3 FP, $A=3.6$, $\beta=15\%$, $|\mathbf{B}|=1$ T, Max. Tor. Cur. = 155 kA
High $\beta$ Case: $|B|$/Flux Surface Alignment

- $|B|$ surfaces align with flux surfaces at higher $\beta$
- Leads to improved omnigeneity/quasi-symmetry

$\beta=0\%$: 

$\beta=23\%$: 

\[ \text{Equilibrium} \]
$|B|$ contours (given at $r/a = 0.75$) show a significant improvement in poloidal symmetry with increasing $\langle \beta \rangle$.

$\langle \beta \rangle = 0\%$

$\langle \beta \rangle = 23\%$
Through its modification of $|B|$, high $\beta$ improves the thermal ion neoclassical confinement time.
Increasing $\beta$ leads to improved neoclassical transport and to a decreased bootstrap current coefficient.
(results shown are for 3 field period, $A = 3.4$ device)
α-particle slowing-down simulations show these devices indicate very good confinement with increasing β.

The configuration was scaled to $\langle B \rangle = 5T$ and $R_0 = 10m$ for alpha confinement studies.
Summary

- Attractive 2 and 3 field period devices have been found for $A = 2.5 - 3.5$
  - Attain good confinement by being near quasi-poloidal symmetry
  - Modular coils: good flux surface reconstruction, preserves physics

- Different heating options and magnetic field variation (0.5 – 1T) allow exploration of different confinement regimes
  - ECH: $\tau_{\text{neo}} / \tau_{\text{ISS95}}$ from 1.4 to 2
  - ICH: $\tau_{\text{neo}} / \tau_{\text{ISS95}}$ from 3 to 3.6

- Quasi-poloidal symmetry minimizes viscous damping in the direction of the $E_r \times B$ drifts
  - lower parallel flows
  - influences accessibility of enhanced confinement regimes which rely on $E_r \times B$ shear
  - potential for more direct manipulation of $E_r \times B$ than in a tokamak
Summary (contd.)

• High $\beta$ configurations offer improved confinement with increasing $\beta$
  – Large fraction of the transform from plasma current
  – Similar to advance tokamak, but bootstrap current is well aligned and not too large (as it is in an axisymmetric device)
  – Stability limits (ballooning, kink, vertical) allow operation at $<\beta> \sim 15\%$ (second regime stability)
  – Have achieved lowest alpha losses (~12\%) of any of our configurations