Ideal MHD Stability of NCSX

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NCSX Physics Validation Review
Princeton, NJ, March 26-28, 2001

Outline

- We present the physics basis for ideal MHD stability in NCSX.

- We have

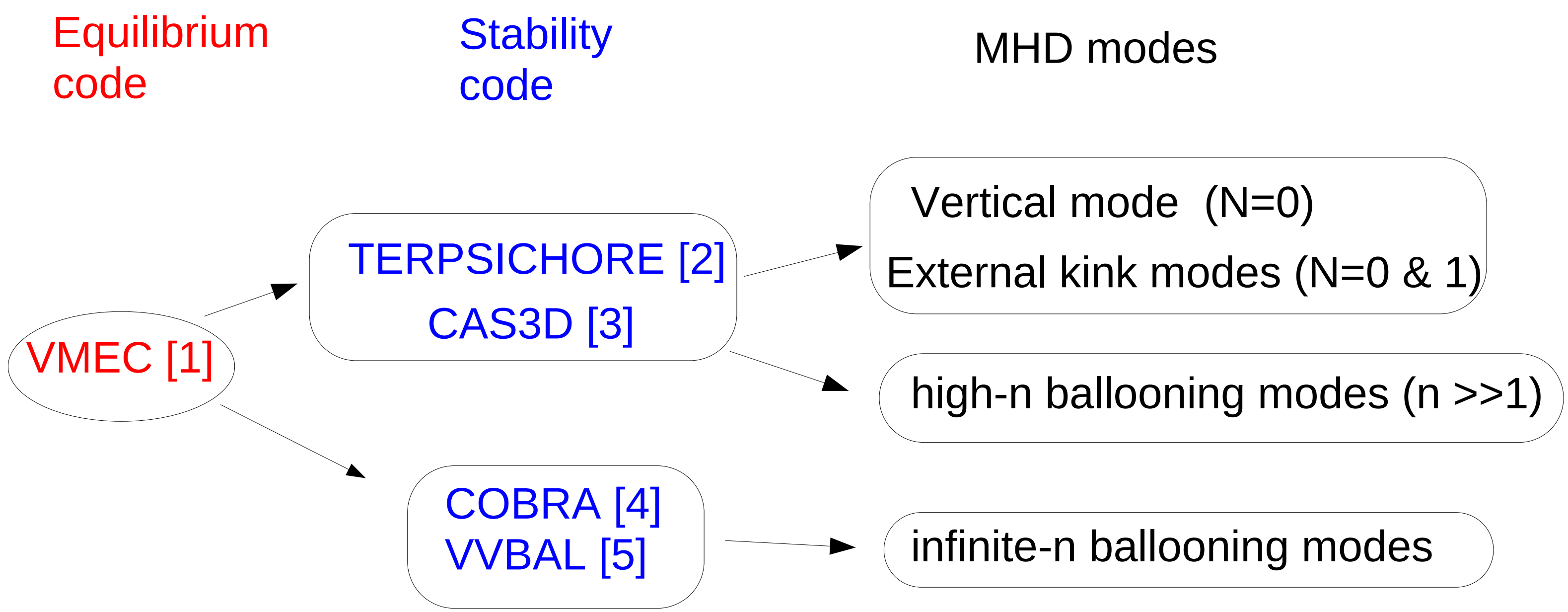
  validated the 3D MHD stability codes used in the design of NCSX;

  achieved a good understanding of stabilization effects of 3D shaping;

  analyzed the MHD stability property of the NCSX configurations.

- High Beta ($\geq 4\%$) free-boundary equilibria of NCSX are found that are stable to ideal MHD modes including ballooning modes, external kink modes, the vertical mode and Mercier modes with good quasi-symmetry.

Stability Codes and Their Validation



[1] Hirshman S. P. and Whitson, J. C., Phys. Fluids **26** (1983) 3553.

[2] Anderson, D. V. *et al.*, Scient. Comp. Supercomputer II, (1990) 159.

[3] Nüehrenberg, C., Phys. Plas. **3**, (1996) 2401.

[4] Sanchez, R. *et al.*, J. Comp. Physics **161**, (2000) 576.

[5] Cooper, W. A., Phys. Plasmas **3**, 275(1996)

- Cobra solves the ideal ballooning mode equation for eigenvalue γ^2:

$$\rho\gamma^2\frac{\mathbf{k}_\perp^2}{B^2}\Phi - \mathbf{B}\cdot\nabla\frac{\mathbf{k}_\perp^2}{B^2}\mathbf{B}\cdot\nabla\Phi - \frac{p'}{B^2}(\mathbf{k}_\perp \times \mathbf{B})\cdot\kappa\Phi = 0 \tag{1}$$

  where $\mathbf{k}_\perp = \nabla\phi - q(\psi)\nabla\theta - q'(\theta - \theta_k)\nabla\psi$.

- Terpsichore-VVBAL solves a modified ballooning mode equation for eigenvalue λ:

$$\mathbf{B}\cdot\nabla\frac{\mathbf{k}_\perp^2}{B^2}\mathbf{B}\cdot\nabla\Phi + (1-\lambda)\frac{p'}{B^2}(\mathbf{k}_\perp \times \mathbf{B})\cdot\kappa\Phi = 0 \tag{2}$$

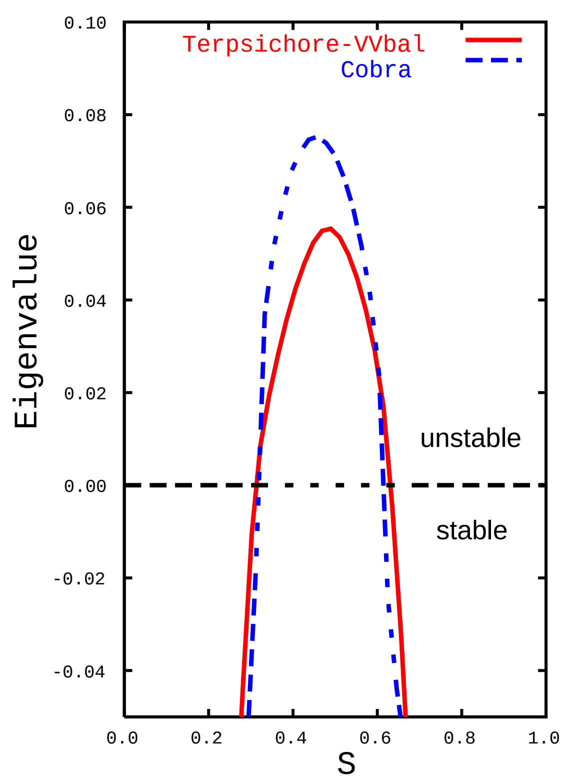
  where $(1-\lambda)$ is an artificial multiplier to the pressure-gradient term and $\lambda > 0$ for instability.

- Note that the eigenvalues are defined differently in two codes but the marginal stability points are the same.

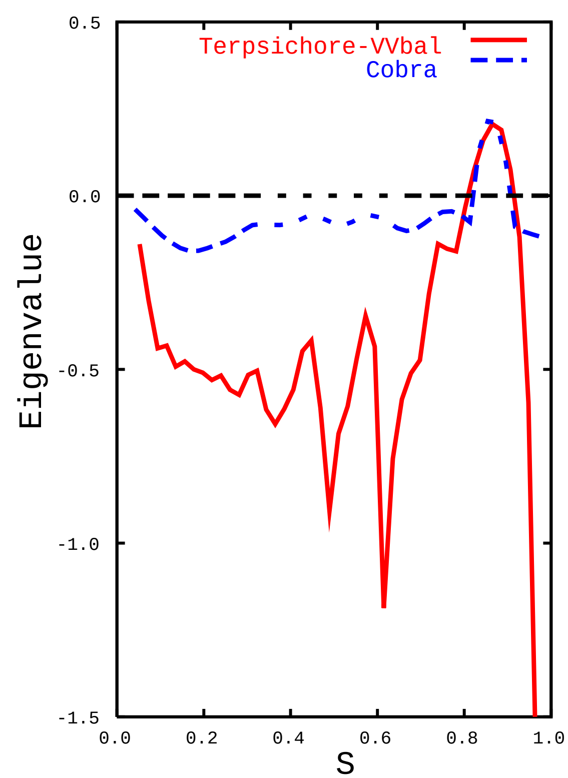
- $\gamma = \gamma(s, \theta_k, \alpha)$ with s being flux label, θ_k being the radial wave number and α being the field line variable.

- The Terpsichore-VVBAL's marginal stability points agree well with those of Cobra. Note that the eigenvalues differ because of the different definitions.

Quasi-helical Case



Preliminary QAS Case



- The Terpsichore and CAS3D are 3D ideal MHD stability codes that determine stability by minimizing the plasma potential energy:

$$\delta W_p = \frac{1}{2}\int d^3x[\delta \mathbf{B}_{\perp}{}^2 + (\delta \mathbf{B}_{\parallel} - \mathbf{B}\frac{\xi \cdot \nabla p}{B^2})^2 + \mathbf{j}_{\parallel} \cdot \xi \times \delta \mathbf{B} - 2\xi \cdot \nabla p \; \xi \cdot \kappa]$$

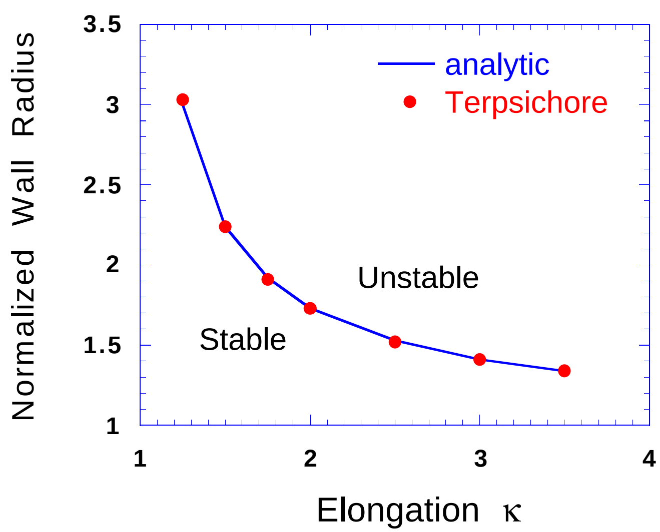
- Both codes use a finite element method for radial discretization and Fourier decomposition in poloidal and toroidal angles.

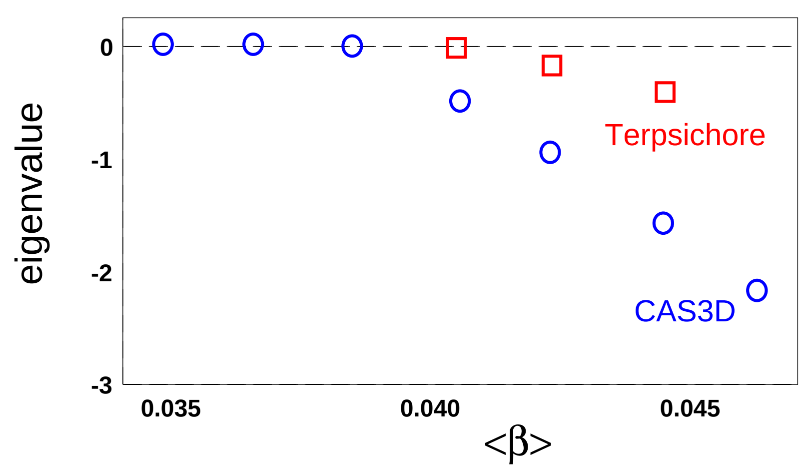
- The Terpsichore treats vaccum as a pseudo-plasma. The CAS3D uses Green's function method to solve the vacuum problem and thus can evaluate stability without a conducting wall.

- The Terpsichore code is used in the optimizer for sake of speed.

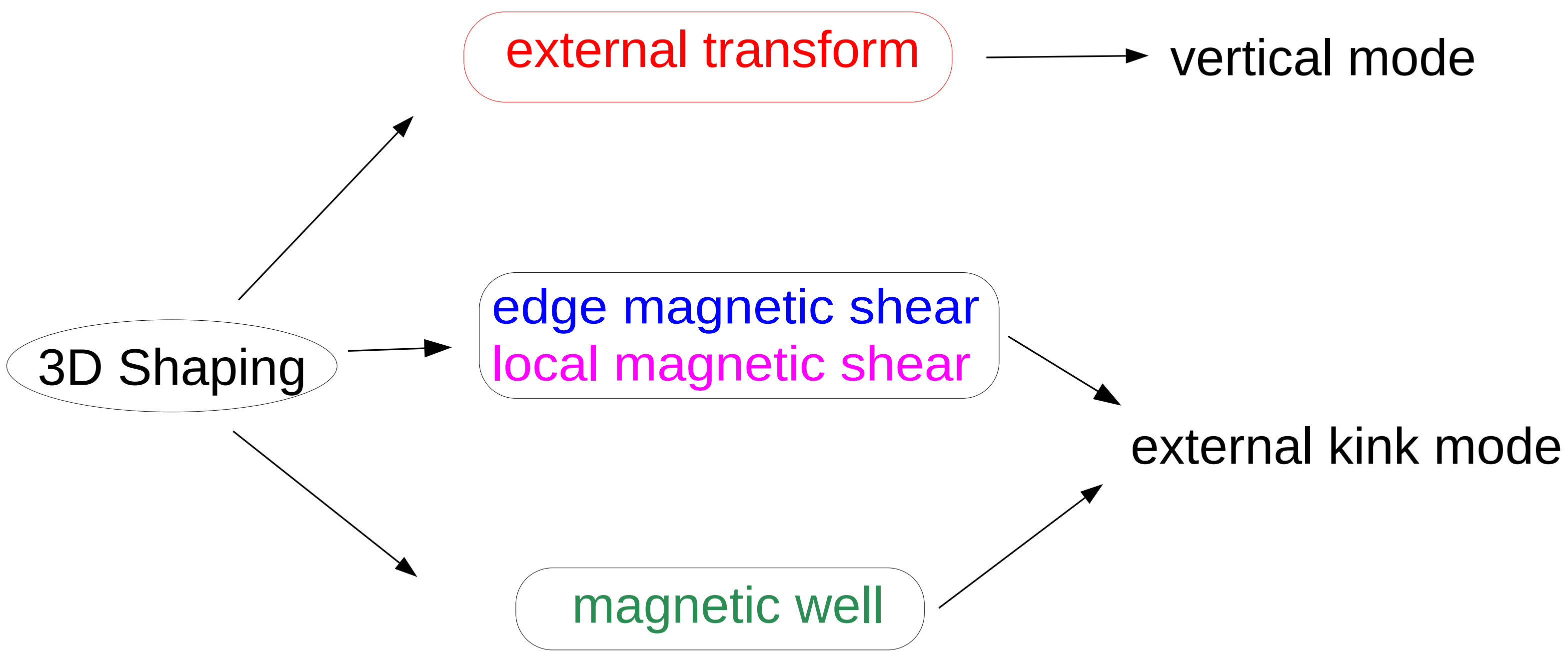
Global Stability Code Benchmark

- The Terpsichore results agree with PEST and CAS3D for the beta limit of an $n = 1$ external kink in an optimized ARIES-RS tokamak.
- The Terpsichore results agree with an analytic stability criterion for the vertical mode (left figure).
- The Terpsichore results agree with CAS3D's for the critical beta of the QAS configuration C82 (right figure).





Stabilization Mechanisms



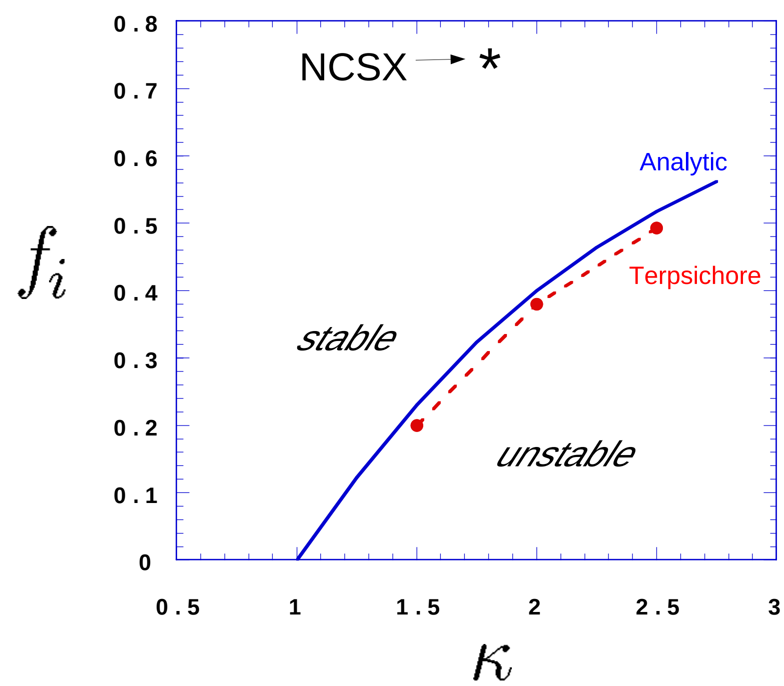
Stabilization of Vertical Mode by External Transform

- In tokamaks, the vertical mode is unstable for elongation $\kappa > 1$;
- in stellarators, the external transform is stabilizing because there is less current to drive the mode for a given amount of total rotational transform;
- We have derived an analytic criterion for the critical external transform assuming $R/a >> 1$, $\beta = 0$, and uniform current profile.

$$f_{i\_crit} = \frac{\kappa^2 - \kappa}{\kappa^2 + 1}$$

f_i : fraction of external transform

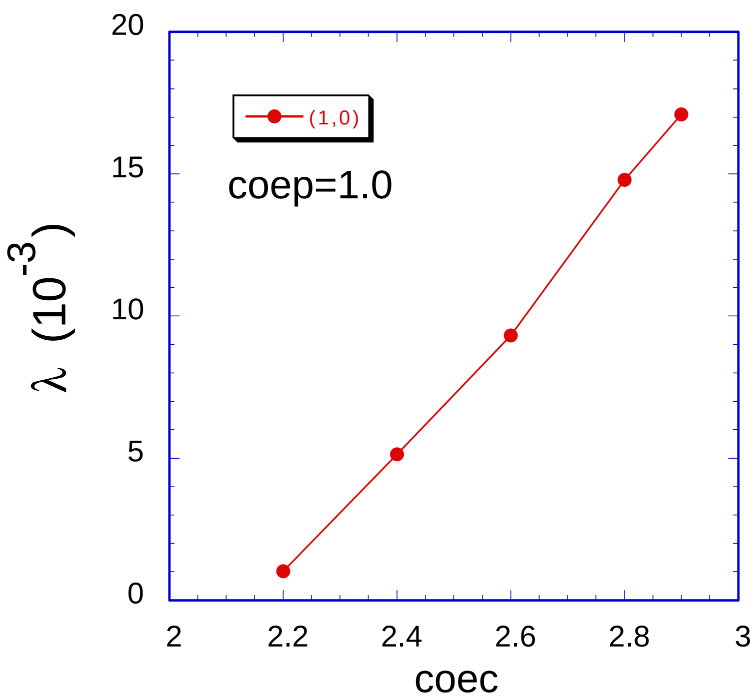
κ : averaged elongation

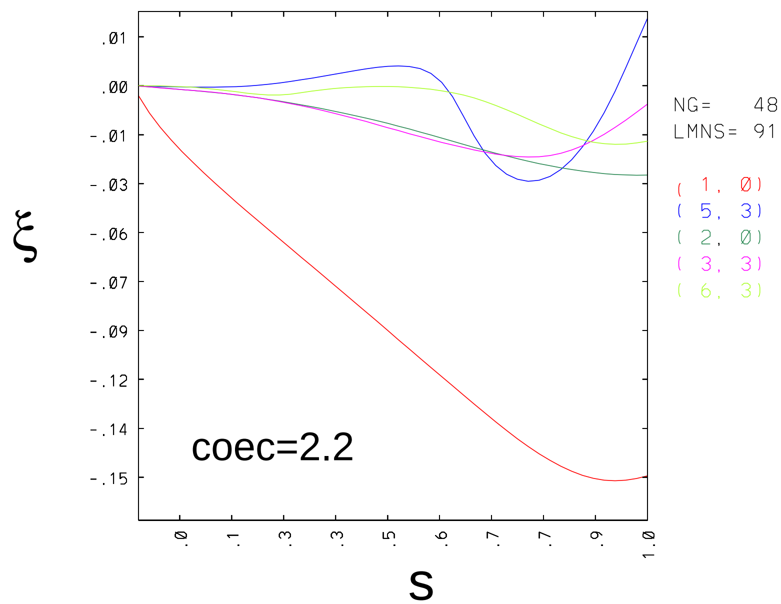


Fu, G. Y. *et al.*, Phys. Plasmas **7**, (2000) 1079.

The Vertical Mode is Robustly Stable

The fixed boundary NCSX reference configuration (LI383) is calculated to be robustly stable to the vertical mode, in agreement with the analytic result.



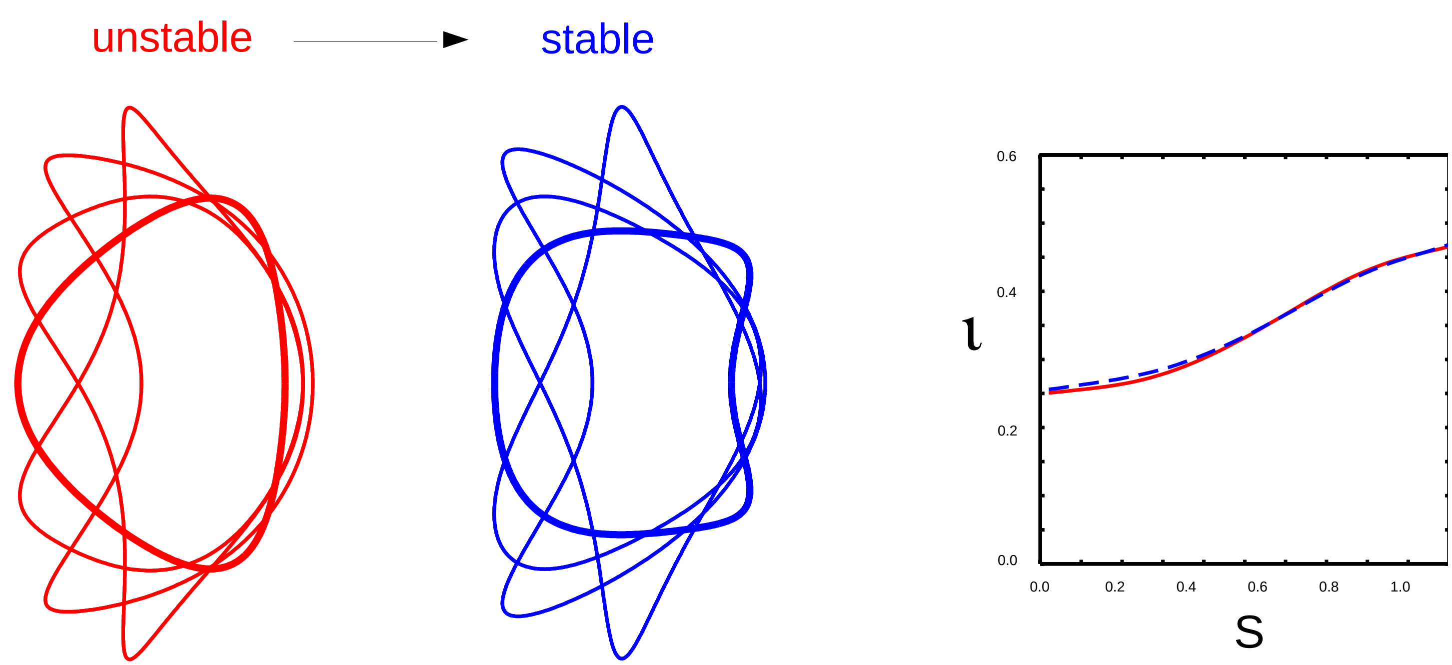


where coec is an artificial multiplier to the parallel current term in the δW:

$$\delta W_p = \frac{1}{2}\int d^3x[\delta \mathbf{B}_\perp{}^2 + (\delta \mathbf{B}_\parallel - \mathbf{B}\frac{\xi \cdot \nabla p}{B^2})^2 + coec\ \mathbf{j}_\parallel \cdot \xi \times \delta\mathbf{B} - coep\ 2\xi \cdot \nabla p\ \xi \cdot \kappa]$$

Stabilization of External Kinks by 3D Shaping at Fixed Iota

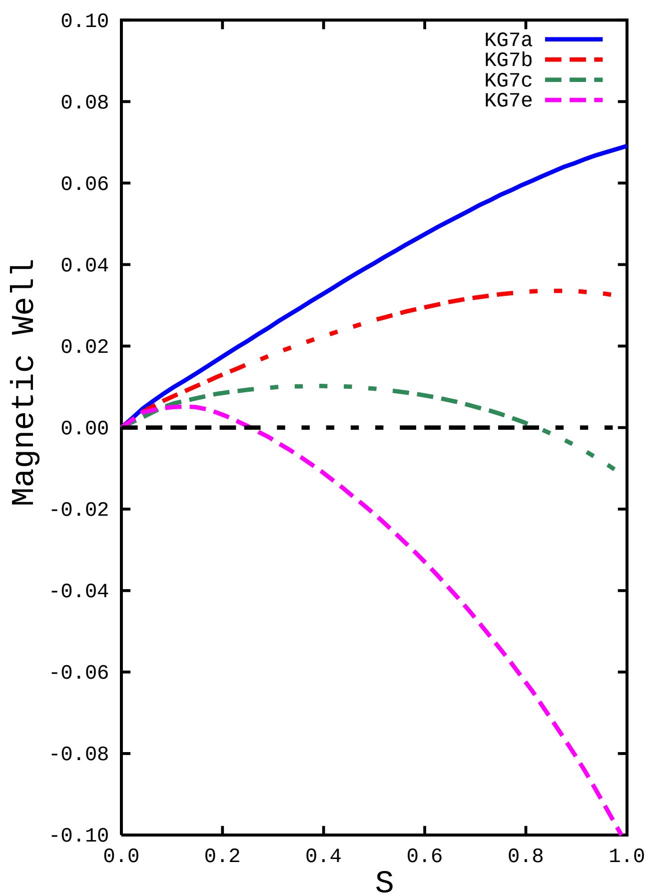
We demonstrate here the stabilization of external kinks due to 3D shaping at fixed iota profile.

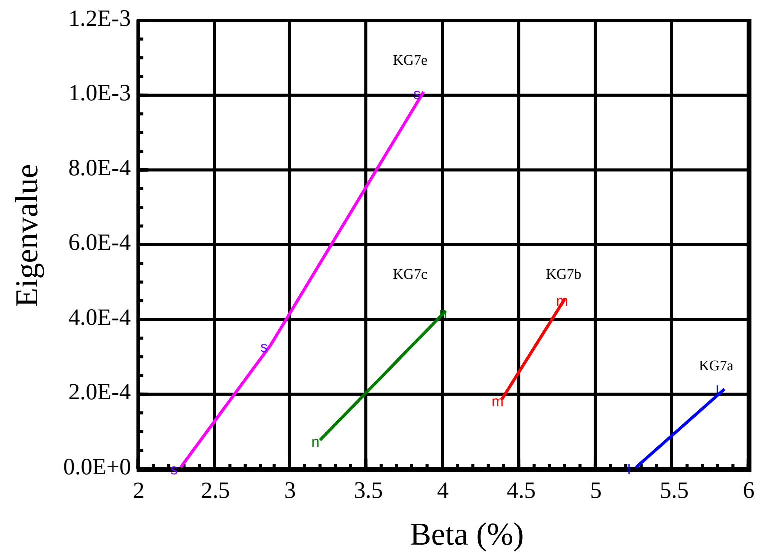


Detailed analysis showed that the stabilization is due to enhancement of local magnetic shear that increases the field line bending energy.

Stabilization due to Magnetic Well

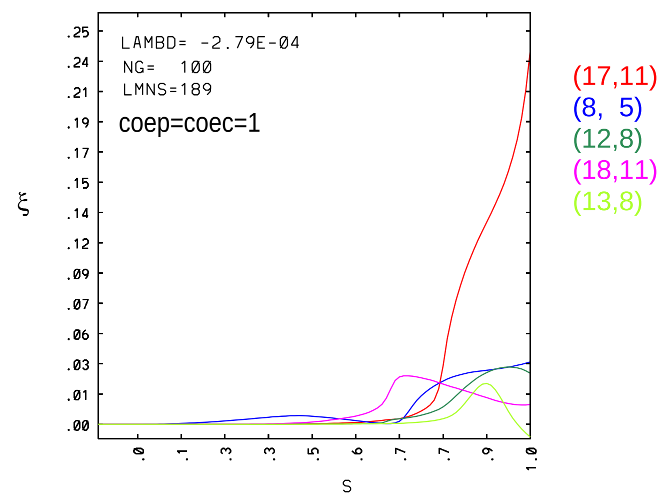
Investigate the effects of vacuum magnetic well on the kink stability for a sequence of QAS equilibria with decreasing magnetic well. The beta limit decreases as the magnetic well decreases.





Stability of NCSX: Fixed Boundary LI383

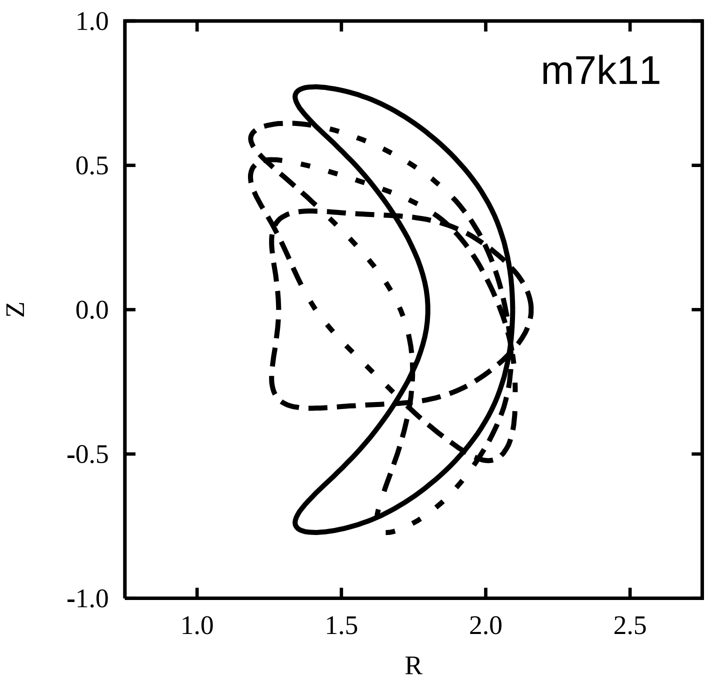
- The NCSX reference configuration LI383 (fixed boundary) was optimized to be marginal stable using reasonable numerical resolutions (typically 49 surfaces and 91 perturbation modes with n up to 7).
- Recent convergence study reveals a weak high-n external kink mode by using much higher resolutions (100 surfaces and 201 perturbation modes with n up to 26). This level of resolution is too costly for configuration optimization.

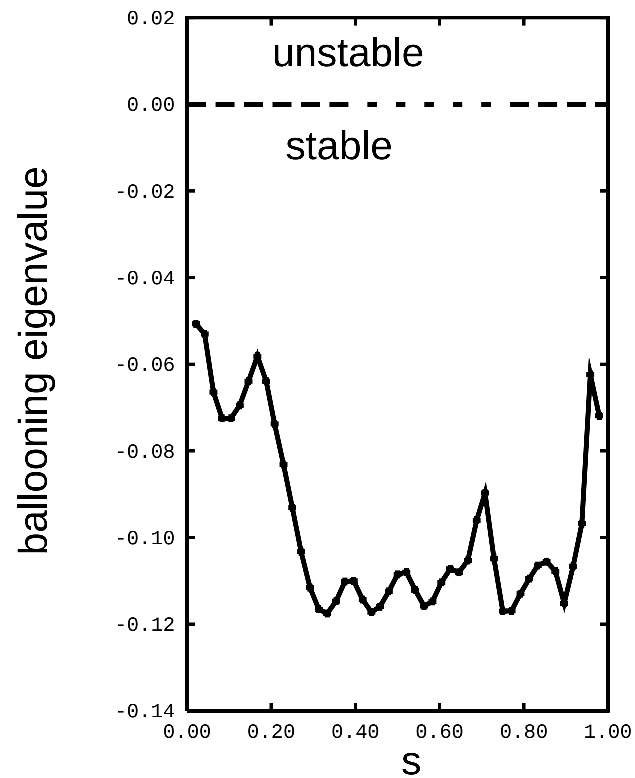


- This high-n mode can be stabilization by a small modification to the 3D shape.

Free Boundary NCSX Configurations are Stable

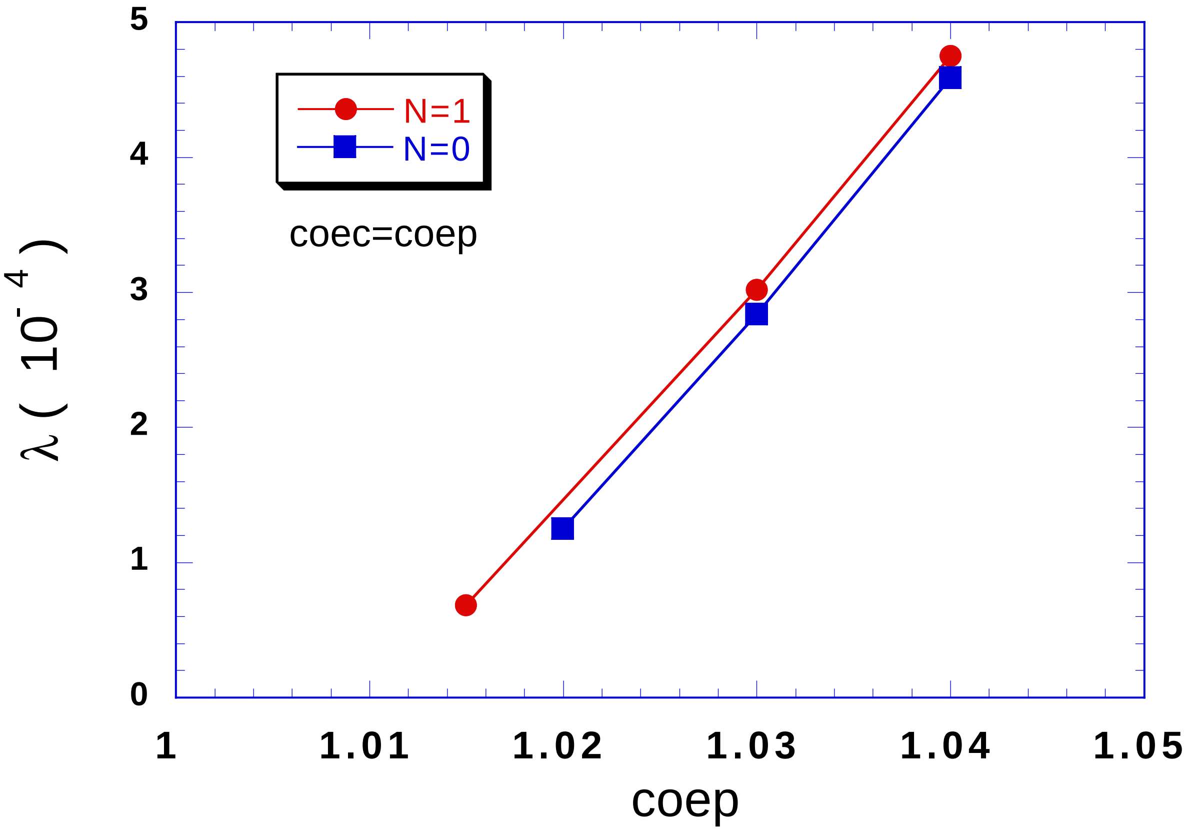
We have found several free boundary configurations from coils that are stable to ballooning modes, external kink and vertical modes. Here is a stable case at $\beta = 4.1\%$ from 1219 (m7) coils.





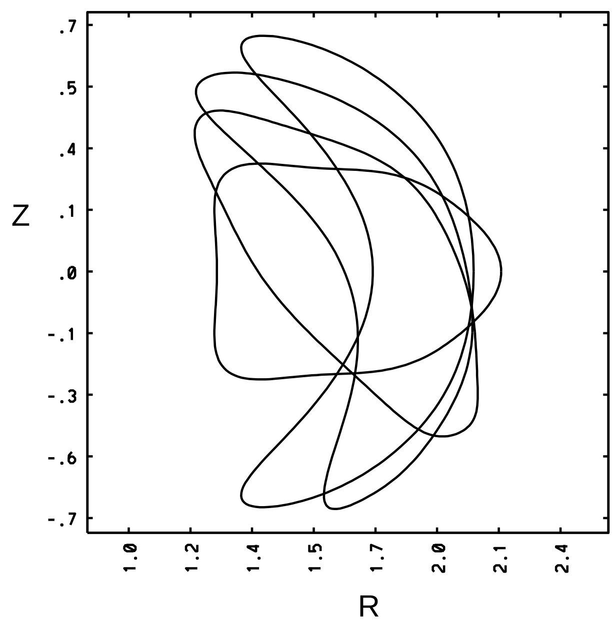
The External Kinks are Stable for the M7 Coils

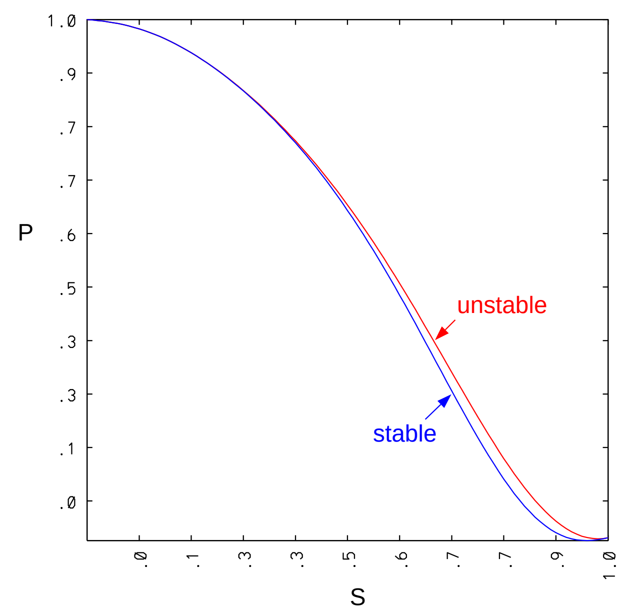
Results are obtained with 100 flux surfaces and 201 perturbation modes at $coec = coec$ where $coec/coep$ is an artificial multiplier to the kink/ballooning term in the δW_p.



A Stable High Beta Configuration from M2 Coils

- A stable high-beta configuration was found from the robustness study in $I_p - \beta$ space. The configuration was recently found to be weakly unstable to a high-n external kink mode using high resolutions and was re-stabilized by locally flattening the pressure profile near the edge. The kink beta limit is $> 6.5\%$.





Conclusions

- The MHD stability of current-carrying compact stellarators is investigated using most advanced MHD codes.

- We have identified physical mechanisms responsible for stabilization of MHD modes by 3D shaping:

  stabilization of the vertical mode due to external rotational transform;

  stabilization of the external kink modes due to magnetic shear and magnetic well.

- We have found several high-beta ($\beta > 4\%$) free boundary configurations of NCSX stable to ballooning modes, external kink and vertical modes, and Mercier modes.